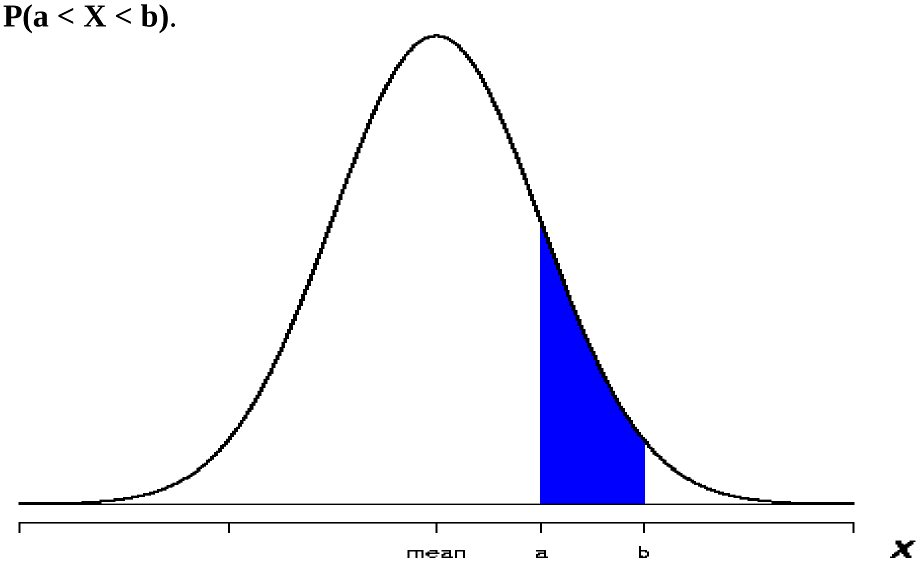
Section 7.5 – The Normal Distribution
Section 7.6 – Application of the Normal Distribution

A random variable that may take on infinitely many values is called a **continuous random variable**.

A continuous probability distribution is defined by a function *f* called the **probability density function**.

The probability that the random variable X associated with a given probability density function assumes a value in an interval $a < x < b$ is given by the area of the region between the graph of *f* and the x-axis from $x = a$ to $x = b$.

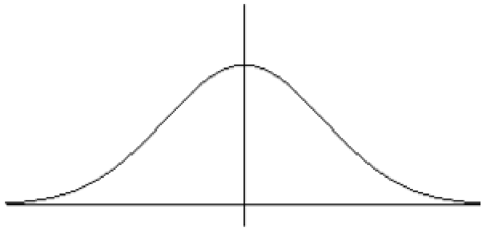
The following graph is a picture of a normal curve and the shaded region is **P(a < X < b)**.



Note: $P(a < X < b) = P(a \leq X < b) = P(a < X \leq b) = P(a \leq X \leq b)$, since the area under one point is 0. The **area of the region under the standard normal curve to the left** of some value z, i.e. $P(Z < z)$ or $P(Z \leq z)$, is calculated for us in the **Standard Normal Cumulative Probability Table** found in Chapter 7 of the online book.

Normal distributions have the following characteristics:

1. The graph is a bell-shaped curve.



The curve always lies above the *x*-axis but approaches the *x*-axis as *x* extends indefinitely in either direction.

2. The curve has peak at $x = \mu$. The mean, μ, determines where the center of the curve is located.

3. The curve is symmetric with respect to the vertical line $x = \mu$.

4. The area under the curve is 1.

5. σ determines the sharpness or the flatness of the curve.

6. For any normal curve, 68.27% of the area under the curve lies within 1 standard deviation of the mean (i.e. between $\mu - \sigma$ and $\mu + \sigma$), 95.45% of the area lies within 2 standard deviations of the mean, and 99.73% of the area lies within 3 standard deviations of the mean.

The Standard Normal Variable will commonly be denoted *Z*. The **Standard Normal Curve** has $\mu=0$ and $\sigma=1$.

Example 1: Let *Z* be the standard normal variable. Find the values of:

a. $P(Z < -1.91)$

b. P(Z > 0.5)

c. P(-1.65 < Z < 2.02)

Example 2: Let Z be the standard normal variable. Find the value of z if z satisfies:

a. P(Z < -z) = 0.9495

b. P(Z > z) = 0.9115

c. P(-z < Z < z) = 0.8444

Formula: $P(Z < z) = \frac{1}{2}\left[1 + P(-z < Z < z)\right]$

When given a normal distribution in which $\mu \neq 0$ and $\sigma \neq 1$, we can transform the normal curve to the standard normal curve by doing whichever of the following applies.

P(X < b) = $P\left(Z < \frac{b-\mu}{\sigma}\right)$

P(X > a) = $P\left(Z > \frac{a-\mu}{\sigma}\right)$

P(a < X < b) = $P\left(\frac{a-\mu}{\sigma} < Z < \frac{b-\mu}{\sigma}\right)$

Example 3: Suppose X is a normal variable with $\mu = 7$ and $\sigma = 4$. Find P(X > -1.35).

Applications of the Normal Distribution

Example 4: The heights of a certain species of plant are normally distributed with a mean of 20 cm and standard deviation of 4 cm. What is the probability that a plant chosen at random will be between 10 and 33 cm tall?

Example 5: Reaction time is normally distributed with a mean of 0.7 second and a standard deviation of 0.1 second. Find the probability that an individual selected at random has a reaction time of less than 0.6 second.

Approximating the Binomial Distribution Using the Normal Distribution

Theorem

Suppose we are given a binomial distribution associated with a binomial experiment involving n trials, each with probability of success p and probability of failure q. Then if n is large and p is not close to 0 or 1, the binomial distribution may be approximated by a normal distribution with $\mu = np$ and $\sigma = \sqrt{npq}$.

Example 6: Consider the following binomial experiment. Use the normal distribution to approximate the binomial distribution. A company claims that 42% of the households in a certain community use their Squeaky Clean All Purpose cleaner. What is the probability that between 15 and 28, inclusive, households out of 50 households use the cleaner?

Example 7: Use the normal distribution to approximate the binomial distribution. A basketball player has a 75% chance of making a free throw. She will make 120 attempts. What is the probability of her making:

a. 100 or more free throws?

b. fewer than 75 free throws?

Example 8: Use the normal distribution to approximate the binomial distribution. A die is rolled 84 times. What is the probability that the number 2 occurs more than 11 times?