50 min
13 Questions
11 M-C 84 pts
2 FR 16 pts

Math 1313
Test 2 Review

1. A piece of equipment was purchased by a company for $10,000 and is assumed to have a scrap value of $3,000 in 5 years. Assume its value is depreciated linearly.

a. Find the expression for the machine's book value in the t-th year of use ($0 \le t \le 5$).

$V(t) = mt + \text{initial}$ $\quad m = \dfrac{\text{scrap value} - \text{initial value}}{\text{time}}$

$m = \dfrac{3000 - 10000}{5} = -1400$

$V(t) = -1400t + 10000$

b. Find the value of the equipment after 3 years.

$V(3) = -1400(3) + 10000 = \$5800$

2. A bicycle manufacturer experiences monthly fixed costs of $175,000 and production costs of $75 per bicycle produced. Each bicycle sells for $125.

a. What is the cost function?

$C(x) = 75x + 175000$

b. What is the revenue function?

$R(x) = 125x$

c. What is the profit function?

$P(x) = R(x) - C(x) = 125x - (75x + 175000)$

d. What is the break-even point? $P(x) = 50x - 175000$

$P(x) = 0$

$50x - 175000 = 0$
$50x = 175000$
$x = 3500$ bikes

$R(3500) = 125(3500)$
$= \$437500$

$(3500, 437500)$

e. What is the profit (loss) when the company produces and sells 5500 bicycles?

$P(5500) = 50(5500) - 175000 = \$100,000$

Profit

3. A manufacturing company makes two types of snowboards, a standard model and deluxe model. Each standard model requires 4 labor-hours in the fabricating department and 1 labor-hour in the finishing department. Each deluxe model requires 6 labor-hours in the fabricating department and 1 labor-hour in the finishing department. The company anticipates a profit of $50 on each standard model and $85 on each deluxe model. If the company has at most 108 labor-hours per day available in the fabricating department and 24 labor-hours per day in the finishing department, how many of each type of snowboard should be manufactured each day to realize a maximum profit? What is the maximum profit? Set up the LPP only. Let x = number of standard models and y = number of deluxe models.

	standard	Deluxe	
Fabr. Dept	4	6	≤ 108
Finishing D.	1	1	≤ 24
Max Profit	50	85	

Max $P = 50x + 85y$

s.t.

$$\begin{cases} 4x + 6y \leq 108 \\ x + y \leq 24 \\ x \geq 0 \\ y \geq 0 \end{cases}$$

4. You can use two types of fertilizer in your orange grove, Best Food and Natural Nutri. Each bag of Best Food contains 8 pounds of nitrogen, 4 pounds of phosphoric acid, and 2 pounds of chlorine. Each bag of Natural Nutri contains 3 pounds of nitrogen, 4 pounds of phosphoric acid and 1 pound of chlorine. You know that the grove needs at least 1,000 pounds of phosphoric acid and at most 400 pounds of chlorine. If you want to minimize the amount of nitrogen added to the grove, how many bags of each type of fertilizer should be used? How much nitrogen will be added? Set up the LPP only. Let x = number of bags of Best Food and y = number of bags of Natural Nutri.

	Best Food	Natural Nutri	
Phosphoric Acid	4	4	≥ 1000
Clorine	2	1	≤ 400
Min Nitrogen	8	3	

Min $N = 8x + 3y$

s.t. $4x + 4y \geq 1000$

$2x + y \leq 400$

$x, y \geq 0$

5. Solve the linear programming problem. Max $P = 4x + 6y$

s.t.

$$2x + 5y \le 20$$
$$x + y \le 7$$
$$x, y \ge 0$$

$2x + 5y \overset{\le}{=} 20$

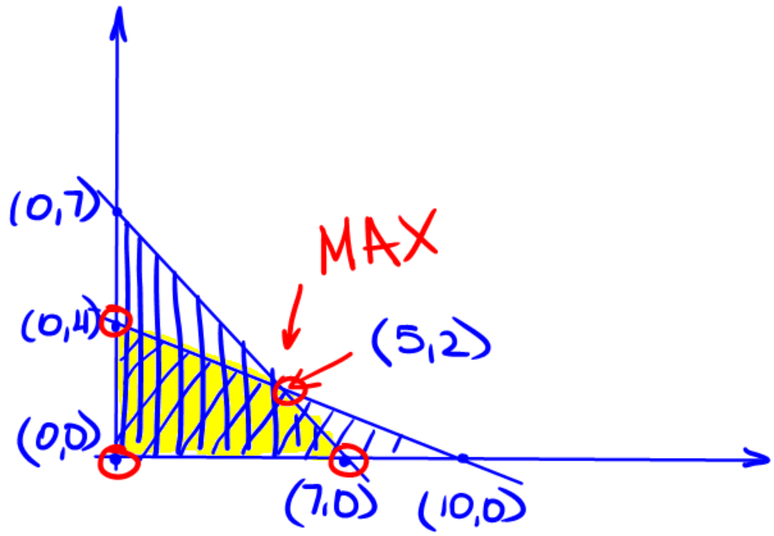
x	y
0	4
10	0

$(0,0)$ $2(0)+5(0) \le 20$ ✓

$x + y \overset{\le}{=} 7$

x	y
0	7
7	0

$(0,0)$ $0+0 \le 7$ ✓



$2x + 5y = 20$

$(x + y = 7) -2$

$$\begin{array}{r} 2x + 5y = 20 \\ + \quad -2x - 2y = -14 \\ \hline 3y = 6 \\ y = 2 \end{array}$$

$x + 2 = 7$

$x = 5$

Max

	$P = 4x + 6y$
$(0,0)$	$4(0) + 6(0) = 0$
$(7,0)$	$4(7) + 6(0) = 28$
$(5,2)$	$4(5) + 6(2) = \boxed{32 \text{ Max}}$
$(0,4)$	$4(0) + 6(4) = 24$

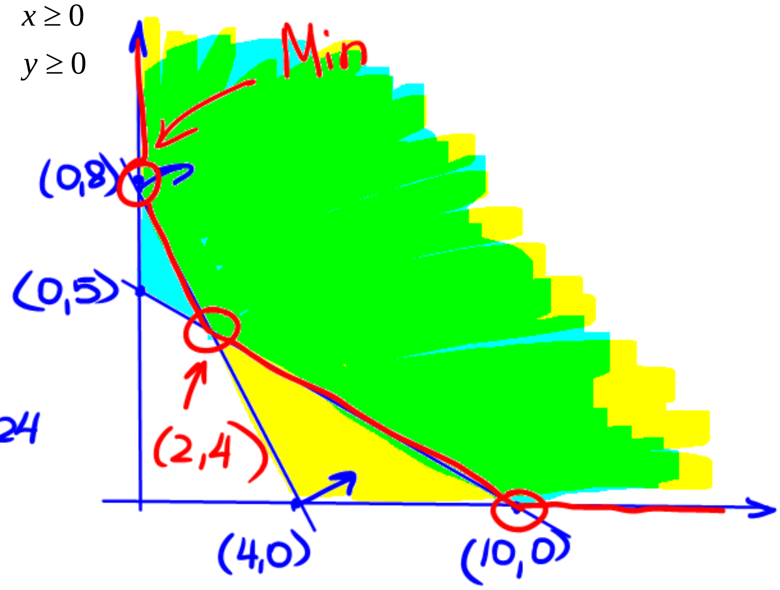
6. Use the method of corners to solve this LPP:
Minimize $Z = 5x + 2y$

subject to $6x + 3y \geq 24$

$3x + 6y \geq 30$

$x \geq 0$

$y \geq 0$



$6x + 3y \overset{\geq}{=} 24$

$2x + y = 8$

x	y
0	8
4	0

$(0,0)$ $6(0) + 3(0) \not\geq 24$

$3x + 6y \overset{\geq}{=} 30$

$x + 2y = 10$

x	y
0	5
10	0

$(0,0)$ $0 + 2(0) \not\geq 10$

$6x + 3y = 24$

$(3x + 6y = 30) - 2$

$$
\begin{aligned}
+\quad 6x + 3y &= 24 \\
-6x - 12y &= -60 \\
\hline
-9y &= -36 \\
y &= 4
\end{aligned}
$$

$6x + 3(4) = 24$

$6x + 12 = 24$

$6x = 12$ $x = 2$

	Min $Z = 5x + 2y$
(10,0)	$5(10) + 2(0) = 50$
(2,4)	$5(2) + 2(4) = 18$
(0,8)	$5(0) + 2(8) = \boxed{16 \text{ Min}}$

FREE RESPONSE: 7. Solve the system of linear equations using the Gauss-Jordan elimination method.

a. $$\begin{aligned} x+3y+z&=3 \\ y+2z&=4 \\ -9y+2z&=-16 \end{aligned}$$

$$\left[\begin{array}{ccc|c} \boxed{1} & 3 & 1 & 3 \\ 0 & \boxed{1} & 2 & 4 \\ 0 & -9 & 2 & -16 \end{array}\right] \quad \begin{aligned} &-3R2+R1 \to R1 \\ &9R2+R3 \to R3 \end{aligned}$$

$$\left[\begin{array}{ccc|c} \boxed{1} & 0 & -5 & -9 \\ 0 & \boxed{1} & 2 & 4 \\ 0 & 0 & \boxed{20} & 20 \end{array}\right] \quad \frac{1}{20}R3 \to R3 \quad \left[\begin{array}{ccc|c} \boxed{1} & 0 & -5 & -9 \\ 0 & \boxed{1} & 2 & 4 \\ 0 & 0 & \boxed{1} & 1 \end{array}\right]$$

$$\begin{aligned} &5R3+R1 \to R1 \\ &-2R3+R2 \to R2 \end{aligned} \quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right] \quad \boxed{\begin{aligned} x&=-4 \\ y&=2 \\ z&=1 \end{aligned}}$$

b. $$\begin{aligned} 2x+3y&=2 \\ x+3y&=-2 \\ x-y&=3 \end{aligned}$$

$$\left[\begin{array}{cc|c} 2 & 3 & 2 \\ 1 & 3 & -2 \\ 1 & -1 & 3 \end{array}\right] \quad R1 \leftrightarrow R2 \quad \left[\begin{array}{cc|c} \boxed{1} & 3 & -2 \\ 2 & 3 & 2 \\ 1 & -1 & 3 \end{array}\right]$$

$$\begin{aligned} &-2R1+R2 \to R2 \\ &-R1+R3 \to R3 \end{aligned} \quad \left[\begin{array}{cc|c} 1 & 3 & -2 \\ 0 & -3 & 6 \\ 0 & -4 & 5 \end{array}\right] \quad \frac{1}{-3}R2 \to R2$$

$$\left[\begin{array}{cc|c} \boxed{1} & 3 & -2 \\ 0 & \boxed{1} & -2 \\ 0 & -4 & 5 \end{array}\right] \quad \begin{aligned} &-3R2+R1 \to R1 \\ &4R2+R3 \to R3 \end{aligned} \quad \left[\begin{array}{cc|c} 1 & 0 & 4 \\ 0 & 1 & -2 \\ 0 & 0 & -3 \end{array}\right]$$

No solution

c. $\begin{aligned} x-2y&=2 \\ 7x-14y&=14 \\ 3x-6y&=6 \end{aligned}$

$$\left[\begin{array}{cc|c} \boxed{1} & -2 & 2 \\ 7 & -14 & 14 \\ 3 & -6 & 6 \end{array}\right] \begin{array}{l} -7R1+R2 \to R2 \\ -3R1+R3 \to R3 \end{array}$$

0

0

$$\left[\begin{array}{cc|c} 1 & -2 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$x-2y=2$

$x=2y+2$

Inf. many solutions.

8. Indicate whether the matrix is in row-reduced form.

a. $\left(\begin{array}{cc|c} \boxed{1} & -2 & 0 \\ 0 & 0 & 0 \end{array}\right)$ YES

b. $\left(\begin{array}{cc|c} \boxed{1} & 1 & -1 \\ 0 & \boxed{1} & 10 \end{array}\right)$ 0 NO

c. $\left(\begin{array}{ccc|c} 1 & 0 & 1 & 9 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 0 & 0 \end{array}\right)$ YES

d. $\left(\begin{array}{ccc|c} \boxed{1} & 0 & 0 & 2 \\ 0 & \boxed{1} & 0 & 5 \\ 0 & -9 & 1 & 1 \end{array}\right)$ 0 NO

e. $\left(\begin{array}{ccc|c} 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -4 \end{array}\right)$ NO

f. $\left(\begin{array}{cc|c} 1 & 0 & -5 \\ 0 & 1 & 11 \\ 0 & 0 & 0 \end{array}\right)$ YES

g. $\left(\begin{array}{ccc|c} 1 & 0 & 3 & 9 \\ 0 & 1 & -1 & 8 \\ 0 & 0 & 0 & 0 \end{array}\right)$ YES

9. The following augmented matrix in row-reduced form is equivalent to the augmented matrix of a certain system of linear equations. Use this result to solve the system of equations.

a. $\left(\begin{array}{cc|c} 1 & -1 & 3 \\ 0 & 3 & 1 \\ 0 & 0 & -\frac{1}{3} \end{array}\right)$

No solution

b. $\left(\begin{array}{ccc|c} 1 & 0 & 1 & -3 \\ 0 & 1 & -1 & -17 \end{array}\right)$

Inf. many

$x+z=-3$

$x=-z+3$

$y-z=-17$

$y=z-17$

$z=$ any real #

c. $\left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -8 \\ 0 & 0 & 1 & 7 \end{array}\right)$

$\boxed{\begin{aligned} x&=1 \\ y&=-8 \\ z&=7 \end{aligned}}$

10. Perform the indicated operations.

a. $-4\begin{pmatrix} 0 & 2 \\ -3 & 3 \\ 2 & 1 \end{pmatrix} + \begin{pmatrix} -6 & 0 \\ -2 & -1 \\ 10 & 12 \end{pmatrix}$

$$= \begin{bmatrix} -4(0)-6 & -4(2)+0 \\ -4(-3)-2 & -4(3)-1 \\ -4(2)+10 & -4(1)+12 \end{bmatrix} = \begin{bmatrix} -6 & -8 \\ 10 & -13 \\ 2 & 8 \end{bmatrix}$$

b. $\begin{pmatrix} 5 & 7 \\ 9 & 5 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} -2 & 8 \\ -3 & 10 \end{pmatrix} - 9\begin{pmatrix} -2 & -1 \\ -6 & -15 \end{pmatrix}$

$$= \begin{bmatrix} 5+\frac{1}{2}(-2)-9(-2) & 7+\frac{1}{2}(8)-9(-1) \\ 9+\frac{1}{2}(-3)-9(-6) & 5+\frac{1}{2}(10)-9(-15) \end{bmatrix}$$

$$\begin{bmatrix} 22 & 20 \\ 61.5 & 145 \end{bmatrix}$$

11. a. Find the value for *y*.

$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ x & -1 \end{bmatrix} - 3\begin{bmatrix} y-1 & 2 \\ 1 & 2 \\ 4 & -3 \end{bmatrix} = 2\begin{bmatrix} -4 & -2 \\ 0 & -1 \\ 4 & 4 \end{bmatrix}$

$1 - 3(y-1) = 2(-4)$

$1 - 3y + 3 = -8$

$4 - 3y = -8$

$-3y = -12$ $\boxed{y = 4}$

b. Find the value of *x*.

$2\begin{pmatrix} x & -2 \\ 3 & y \end{pmatrix} - \begin{pmatrix} -2 & z \\ -1 & 2 \end{pmatrix} = 10\begin{pmatrix} 4 & -2 \\ 2u & 4 \end{pmatrix}$

$2x - (-2) = 10(4)$

$2x + 2 = 40$

$2x = 38$

$\boxed{x = 19}$

$a_{ij} \to a_{ji}$

12. Find the transpose of matrix of each matrix.

a. $A = \begin{pmatrix} 0 & -7 \\ -5 & 9 \\ 3 & 1 \end{pmatrix}$ b. $B = \begin{pmatrix} -3 & 10 & 11 \\ 4 & -7 & -4 \\ 3 & 0 & 8 \end{pmatrix}$

$A^T = \begin{bmatrix} 0 & -5 & 3 \\ -7 & 9 & 1 \end{bmatrix}$ $B^T = \begin{bmatrix} -3 & 4 & 3 \\ 10 & -7 & 0 \\ 11 & -4 & 8 \end{bmatrix}$

13. Multiply, if possible.

a. $\begin{pmatrix} 1 & -2 \\ -7 & 10 \end{pmatrix}\begin{pmatrix} 2 & -4 \\ 9 & -6 \end{pmatrix} = \begin{bmatrix} 1(2)-2(9) & 1(-4)-2(-6) \\ -7(2)+10(9) & -7(-4)+10(-6) \end{bmatrix}$

$(2\times 2)(2\times 2)$

$\begin{bmatrix} -16 & 8 \\ 76 & -32 \end{bmatrix}$

b. $\begin{pmatrix} 1 & 4 \\ -6 & 2 \\ -3 & 10 \end{pmatrix}\begin{pmatrix} 1 & 3 \\ -4 & -2 \end{pmatrix} = \begin{bmatrix} 1(1)+4(-4) & 1(3)+4(-2) \\ -6(1)+2(-4) & -6(3)+2(-2) \\ -3(1)+10(-4) & -3(3)+10(-2) \end{bmatrix}$

$(3\times 2)(2\times 2)$

$\begin{bmatrix} -15 & -5 \\ -14 & -22 \\ -43 & -29 \end{bmatrix}$

c. $\begin{pmatrix} -4 & -11 & 10 \\ 6 & -7 & 3 \end{pmatrix}\begin{pmatrix} 9 & -7 & 6 & -4 & 11 \\ 15 & -3 & -6 & -20 & 1 \end{pmatrix}$

$(2\times 3)(2\times 5)$ Not possible

14. Find the inverse of the given matrix, if it exists.

a. $C = \begin{pmatrix} 3 & -3 \\ 2 & -2 \end{pmatrix}$

$D = 3(-2) - (-3)(2)$
$= 0$

Singular (no inverse)

b. $B = \begin{pmatrix} -3 & 3 \\ 1 & 3 \end{pmatrix}$

$D = -3(3) - 3(1) = -9 - 3 = -12 \neq 0$

$B^{-1} = \frac{1}{-12}\begin{bmatrix} 3 & -3 \\ -1 & -3 \end{bmatrix} = \begin{bmatrix} \frac{-1}{4} & \frac{1}{4} \\ \frac{1}{12} & \frac{1}{4} \end{bmatrix}$

FREE RESPONSE: 15. Solve the system of linear equations using the inverse of the coefficient matrix.

a. $8x + 5y = 70$
$-x - 5y = 35$

$\begin{bmatrix} 8 & 5 \\ -1 & -5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 70 \\ 35 \end{bmatrix}$

$D = 8(-5) - 5(-1) = -40 + 5 = -35$

$A^{-1} = -\frac{1}{35}\begin{bmatrix} -5 & -5 \\ 1 & 8 \end{bmatrix} = \begin{bmatrix} \frac{1}{7} & \frac{1}{7} \\ -\frac{1}{35} & -\frac{8}{35} \end{bmatrix}$

$A^{-1}B = \begin{bmatrix} \frac{1}{7} & \frac{1}{7} \\ -\frac{1}{35} & -\frac{8}{35} \end{bmatrix}\begin{bmatrix} 70 \\ 35 \end{bmatrix} = \begin{bmatrix} \frac{1}{\cancel{7}}(\cancel{70}^{10}) + \frac{1}{\cancel{7}}(\cancel{35}^{5}) \\ -\frac{1}{\cancel{35}}(\cancel{70}^{2}) - \frac{8}{\cancel{35}}(\cancel{35}) \end{bmatrix} = \begin{bmatrix} 15 \\ -10 \end{bmatrix}$

$\boxed{\begin{matrix} x = 15 \\ y = -10 \end{matrix}}$

b. $2x + 3y = 5$
$3x + 5y = 8$

$\begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$

$D = 2(5) - 3(3) = 1$

$A^{-1} = \begin{bmatrix} 5 & -3 \\ -3 & 2 \end{bmatrix}$

$A^{-1}B = \begin{bmatrix} 5 & -3 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 5 \\ 8 \end{bmatrix} = \begin{bmatrix} 5(5) + (-3)(8) \\ -3(5) + 2(8) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\boxed{\begin{matrix} x = 1 \\ y = 1 \end{matrix}}$

Formulas to be Provided on Test 2

It will be a link.

$C(x) = cx + F$

$R(x) = sx$

$P(x) = R(x) - C(x)$

If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and $D = ad - bc \neq 0$ then $A^{-1} = \frac{1}{D}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$.