14 Questions
4 - 5pts
10 - 8pts

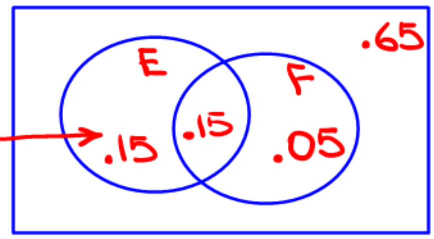
Math 1313
Test 4 Review

1. Let E and F be two events with $P(E)=0.3$ and $P(F)=0.2$ and $P(E\cup F)=0.35$. Find:

a. $P(E\,|\,F)$

$= \dfrac{P(E\cap F)}{P(F)} = \dfrac{.15}{.2} = \boxed{.75}$

$\begin{array}{r} 0.35 \\ -\ .2 \end{array}$



b. $P(F\,|\,E^c)$ ← F only

$= \dfrac{P(F\cap E^c)}{P(E^c)} = \dfrac{.05}{1-0.3} = \dfrac{.05}{.7} = \boxed{.0714}$

2. Companies A, B. and C produce 10%, 40% and 50% respectively of a certain product. It has been found that 1 % from A , 1½ % from B and 2% from C are defective. One of these products is chosen at random.

a. Find the probability the product is defective. $P(D)$

$.1(.01)+.4(.015)+.5(.02)=.017$

b. Find the probability the product is defective and that it was produced by Company C. ($\cap$)

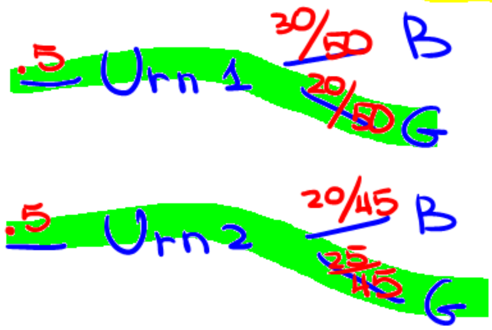
$P(D\cap C)=.5(.02)=.01$

c. Find the probability that it produced by Company C, given it was defective.

$P(C\,|\,D)=\dfrac{P(C\cap D)}{P(D)}=\dfrac{0.01 \leftarrow \text{from b)}}{.017 \leftarrow \text{from a)}}=.5882$

50 45

3. Urn 1 contains 30 blue and 20 green marbles. Urn 2 contains 20 blue and 25 green marbles. An urn is chosen at random with equally likely probability, then a marble is chosen.



a. What is the probability that the marble chosen was green? $P(G)$

$$.5(20/50) + .5(25/45) = \boxed{.4778}$$

b. What is the probability Urn 1 was chosen, given the marble was green?

$$P(\text{Urn1}|G) = \frac{P(\text{Urn1} \cap G)}{P(G)} = \frac{.5(20/50)}{.4778} = \boxed{.4186}$$

4. The odds for rain tomorrow are 2:3. What is the probability it will not rain?

$$\frac{3}{2+3} = \frac{3}{5} = .6$$

5. The probability of an event occurring is 0.4. What are the odds the event will occur?

$P(\text{event}) = .4$

$P(\text{no event}) = .6$

$$\frac{P(\text{event})}{P(\text{no event})} = \frac{.4}{.6} = \frac{4}{6} = \frac{2}{3}$$

2 to 3

2:3

6. A 45 point quiz was given to a history class. The scores are listed below with the corresponding probability. Find the average for this class.

X	P(X=x)
30	0.15
32	0.225
33	0.175
37	0.3
42	0.1

$$\mu(X) = 30(.15) + 32(.225) + 33(.175) + 37(.3) + 42(.1)$$

$$= \boxed{32.775}$$

$\mu = 6.7$ σ

7. The following probability distribution has expected value of 6.7. Find the standard deviation.

X	P(X=x)
3	0. 4
5	0.2
9	0.1
12	0.3

$$Var = .4(3-6.7)^2 + .2(5-6.7)^2 + 0.1(9-6.7)^2 + 0.3(12-6.7)^2 = 15.01$$

$$\sigma = \sqrt{Var} = \sqrt{15.01} = \boxed{3.8743}$$

8. The heights of women who participated in a recent survey have a mean of 64 inches and a standard deviation of 2 inches. Use Chebychev's Inequality to estimate the probability that a woman chosen at random height will be between 60 and 68.

$$P(60 \le X \le 68) \ge 1 - \frac{1}{2^2} = 1 - \frac{1}{4} = \frac{3}{4} = \boxed{.75}$$

$$P(\mu - k\sigma \le X \le \mu + k\sigma) \ge 1 - \frac{1}{k^2}$$

$60 = 64 - k(2)$ $\quad$ $68 = 64 + k(2)$

$k = 2 \longleftrightarrow k = 2$

9. Consider the following binomial experiment. The probability that a person will get a cold this winter is 0.55. Ten people are chosen random. $n = 10$ $p = .55$ $q = .45$

a. Find the probability that is at least 9 people will get a cold.

9 or 10

$$C(10,9)(.55)^9(.45)^1 + C(10,10)(.55)^{10}(.45)^0 = \boxed{.0233}$$

b. Find the probability that is exactly 5 people will get a cold.

$$C(10,5)(.55)^5(.45)^5 = \boxed{.2340}$$

2 or 3 or .. or 10 $\qquad$ 0 or 1

c. Find the probability that at least 2 people will get a cold.

$C(n,x)\,p^x q^{n-x}$

$$1 - \left[C(10,0)(.55)^0(.45)^{10} + C(10,1)(.55)^1(.45)^9\right] = .9955$$

d. Find the mean, variance and standard deviation of the experiment.

$\mu = np = 10(.55) = 5.5$

$Var = npq = 10(.55)(.45) = 2.475$

$\sigma = \sqrt{npq} = \sqrt{10(.55)(.45)} = 1.5732$

10. Let Z be a standard normal random variable. Find the following probabilities:

a. $P(Z < -1.47) = \boxed{.0708}$

Table

b. $P(Z > -1.84) = P(Z < 1.84) = \boxed{.9671}$

Table

c. $P(1.1 < Z < 2.13) = P(Z < 2.13) - P(Z < 1.1)$

Table

$= .9834 - .8643 = \boxed{.1191}$

d. $P(Z < z) = 0.8264$

Table $z = .94$

e. $P(Z > z) = 0.8665$

$P(Z < z) = 1 - 0.8665 = .1335$ $\boxed{z = -1.11}$

Table

f. $P(-z < Z < z) = 0.8690$

$P(Z < z) = \frac{1}{2}(1 + .8690) = .9345$ $\boxed{z = 1.51}$

Table

11. The heights of a certain plant are normally distributive with a mean of 10 inches and a standard deviation of 2 inches. Find the probability that a plant selected at random measures between 8 and 12.

μ σ

$P(8 \le X \le 12)$

$= P\left(\frac{8-10}{2} \le Z \le \frac{12-10}{2}\right)$

$= P(-1 \le Z \le 1) = P(-1 < Z < 1)$

$= P(Z < 1) - P(Z < -1)$

Table

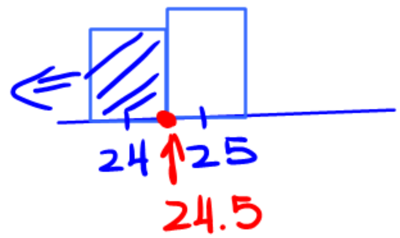
$= .8413 - .1587 = \boxed{.6826}$

12. Use the normal distribution to approximate the binomial distribution. A marksman's chance of hitting a bulls-eye with each of his shots is 82%. If he fires 30 shots, what is the probability of his hitting the target fewer than 25 times?

$n = 30 \quad p = .82 \quad q = .18 \leftarrow 1-p$

$\mu = np = 30(.82) = 24.6$

$\sigma = \sqrt{npq} = \sqrt{30(.82)(.18)} = 2.1043$



$P(X < 25)$

$= P(Y \leq 24.5)$

$= P(Y < 24.5)$

$= P\left(Z < \dfrac{24.5 - 24.6}{2.1043}\right)$

(24.5-24.6)/sqrt(30*.82*.18)

$= P(Z \leq -0.05) = \boxed{.4801}$

Table

Formulas to be Provided on Test 4 and the Z-table will also be provided.

They will be links.

$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A \cap B) = P(B) \cdot P(A \mid B)$$

Given A and B are independent $P(A \cap B) = P(A) \cdot P(B)$

$$E(X) = x_1 p_1 + x_2 p_2 + \ldots + x_n p_n$$

$$\frac{P(E)}{P(E^c)}$$

$$\frac{P(E^c)}{P(E)}$$

If the odds in favor of an event E occurring are *a* to *b*, then the probability of E occurring is

$$P(E) = \frac{a}{a+b}$$

$$Var(X) = p_1(x_1 - \mu)^2 + p_2(x_2 - \mu)^2 + \ldots + p_n(x_n - \mu)^2$$

$$\sigma = \sqrt{Var(X)}$$

$$P(\mu - k\sigma \le X \le \mu + k\sigma) \ge 1 - \frac{1}{k^2}.$$

$$P(X = x) = C(n,x)p^x q^{n-x}$$

$$\mu = np$$

$$Var(X) = npq$$

$$\sigma = \sqrt{Var(X)}$$

$$\text{P}(X < b) = P\left(Z < \frac{b-\mu}{\sigma}\right)$$

$$\text{P}(X > a) = P\left(Z > \frac{a-\mu}{\sigma}\right)$$

$$\text{P}(a < X < b) = P\left(\frac{a-\mu}{\sigma} < Z < \frac{b-\mu}{\sigma}\right)$$

$$P(Z < z) = \frac{1}{2}\left[1 + P(-z < Z < z)\right]$$