Math 1314 Final Exam Review L2-L13

1. The following table of values gives a company's annual profits in millions of dollars. Rescale the data so that the year 2003 corresponds to x = 0.

list

	0	1	2	3	4	5
Year	2003	2004	2005	2006	2007	2008
Profits (in millions of dollars)	31.3	32.7	31.8	33.7	35.9	36.1

*

Begin by creating a list.

a. Find the cubic regression model for the data.

Command: Fitpoly[list1,3]

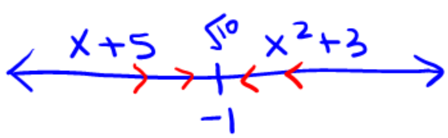
Answer: $f(x) = -0.0556x^3 + 0.531x^2 - 0.3183x + 31.5952$

b. Use the cubic regression model to predict the company's profits in 2010. → $x = 7$

Command: $f(7)$

Answer: \$36.3286 million

2. Suppose $f(x) = \begin{cases} x+5, & x < -1 \\ \sqrt{10} & x = -1 \\ x^2+3, & x > -1 \end{cases}$



Determine, if they exist,

a. $\lim\limits_{x \to -1^-} f(x)$

$= -1+5$

$= \boxed{4}$

b. $\lim\limits_{x \to -1^+} f(x)$

$= (-1)^2+3$

$= \boxed{4}$

c. $\lim\limits_{x \to -1} f(x)$

$= \boxed{4}$

3. $\lim\limits_{x \to 1} \dfrac{\sqrt{4x}}{x-5} = \dfrac{\sqrt{4(1)}}{1-5} = \dfrac{2}{-4} = \boxed{-1/2}$

4. $\lim\limits_{x \to 4} \dfrac{x+3}{2x-8} = \dfrac{4+3}{2(4)-8} = \dfrac{7}{0}$ $\boxed{\text{DNE}}$

5. $\lim\limits_{x \to -2} \dfrac{x^2+5x+6}{x+2} = \dfrac{(-2)^2+5(-2)+6}{-2+2} = \dfrac{0}{0}$

Command: limit[f,-2]

Answer: 1

Rules for limits at infinity KNOW These!

Recall:

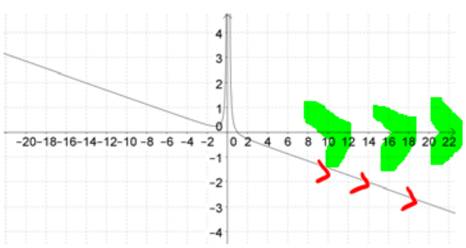
- If the degree of the numerator is smaller than the degree of the denominator, then $\lim\limits_{x\to\infty}\dfrac{f(x)}{g(x)} = 0$.
- If the degree of the numerator is the same as the degree of the denominator, then you can find $\lim\limits_{x\to\infty}\dfrac{f(x)}{g(x)}$ by making a fraction from the leading coefficients of the numerator and denominator and then reducing to lowest terms.
- If the degree of the numerator is larger than the degree of the denominator, then it's best to work the problem viewing the graph in GGB. You can then decide if the function approaches ∞ or $-\infty$. This limit does not exist, but the ∞ or $-\infty$ is more descriptive.

6. $\lim\limits_{x\to\infty}\dfrac{10x^2 - x}{3-4x^2} = \dfrac{10}{-4} = \boxed{-5/2}$

7. $\lim\limits_{x\to -\infty}\dfrac{x^3 + 5x^2 - 7x - 1}{2 + x^2 - 7x^4} = \boxed{0}$

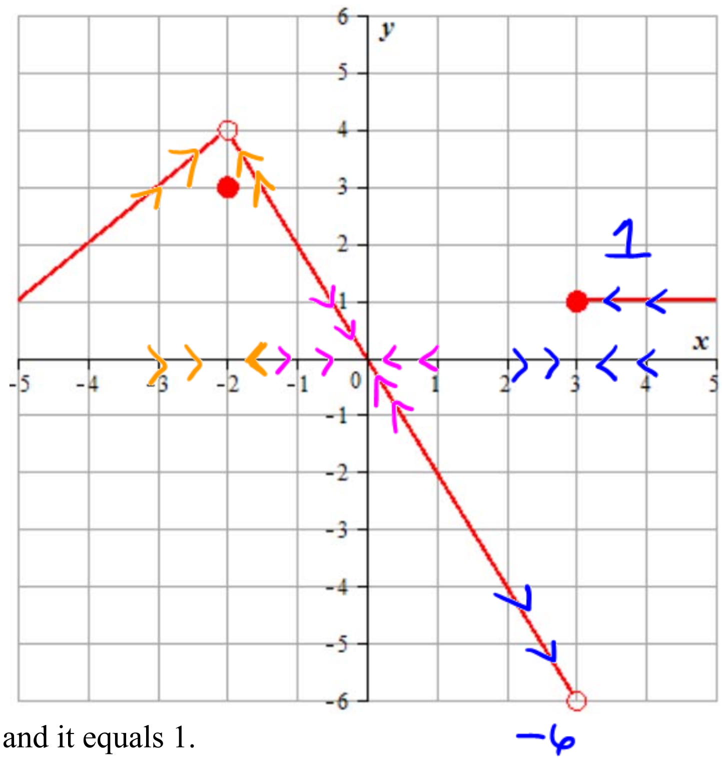
8. $\lim\limits_{x\to\infty}\dfrac{x^3 - 1}{x - 7x^2}$ $\boxed{\text{DNE}}$ or $\boxed{-\infty}$

Enter the function into GGB.



or limit[f, ∞]

9. The graph of $f(x)$ is shown below. Which of the following statements is true?



a. $\lim_{x \to 3} f(x)$ ~~exists~~ and it equals 1.

b. $\lim_{x \to 0} f(x)$ ~~does~~ not exists.

c. $\lim_{x \to -2} f(x)$ exists and it ~~equal 3~~.

d. $\lim_{x \to 3} f(x)$ ~~exists~~ and it equals -6.

e. $\boxed{\lim_{x \to -2} f(x) \text{ exists and it equal 4.}}$

10. Find the first and second derivative: $f(x) = 5x^4 - 3x^3 + 8x^2 + 7x - 1$

$f'(x) = 20x^3 - 9x^2 + 16x + 7$

$f''(x) = 60x^2 - 18x + 16$

11. Let $f(x) = 3x^4 - 5x^2 + 7x - 1$. Find the equation of the tangent line at x = 1.

Recall: tangent[<x-value>, <function>]

Command:

tangent[1, f]

Answer:

$y = 9x - 5$

12. A ball is thrown upwards from the roof of a building at time t = 0. The height of the ball in feet is given by $h(t) = -16t^2 + 148t + 78$, where t is measured in seconds. Find the velocity of the ball after 3 seconds. $t = 3$

Command: $h'(3)$

Answer: 52 ft/s

13. Suppose a manufacturer has monthly fixed costs of $250,000 and production costs of $24 for each item produced. The item sells for $38. Assume all functions are linear. State the:

a. cost function.

$C(x) = cx + F = 24x + 250000$

b. revenue function.

$R(x) = sx = 38x$

c. profit function.

$P(x) = R(x) - C(x) = 38x - (24x + 250000) = 14x - 250000$

d. Find the break-even point.

$R(x) = C(x)$

intersect[R,C]

(17857, $678,566.09)

14. At the beginning of an experiment, a researcher has 569 grams of a substance. If the half-life of the substance is 12 days, how many grams of the substance are left after 21 days?

Begin by making a list.

(0,569) + (12,569/2) ⇒ list1

Command: fitexp[list1]

Answer: $f(x) = 569e^{-0.0578x}$

15. A clothing company manufactures a certain variety of ski jacket. The total cost of producing x ski jackets and the total revenue of selling x ski jackets are given by the following equations

$$C(x) = 27{,}000 + 22x - 0.25x^2$$
$$R(x) = 500x - 0.1x^2$$
$$0 \le x \le 1{,}000$$

Begin by entering the cost and revenue functions into GGB.

a. Find the profit function.

Recall: P(x) = R(x) – C(x)

$P(x) = 0.15x^2 + 478x - 27000$

b. Find the marginal profit function.

Command: $P'(x)$

Answer: $P'(x) = 0.3x + 478$

c. Use the marginal profit to approximate the actual profit realized on the sale of the 201st ski jacket.

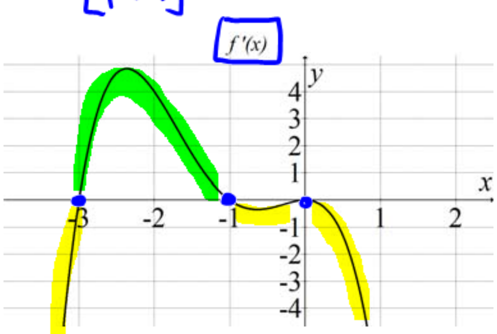
Command: $P'(200)$

Answer: \$538

16. Let $f(x) = -0.2x^5 - x^4 - x^3 - 5$. *Enter the function in GGB.*

a. Find any critical numbers of f.

Commands: root[f'(x)]



Answer:
(-3, 0)
(-1, 0)
(0, 0)

$\boxed{x = -3, -1, 0}$

b. Interval(s) on which f is increasing; interval(s) on which f is decreasing.

f: ↘ ↗ ↘ ↘

f': − + − −

-3 -1 0

Increasing: $(-3, -1)$

Decreasing: $(-\infty, -3) \cup (-1, \infty)$

c. Coordinates of any relative extrema.

Command:

extremum[f]

Answer:

(−3, −10.4) Rel min

(−1, −4.8) Rel max

d. Interval(s) on which f is concave upward; interval(s) on which f is concave downward.

Commands:

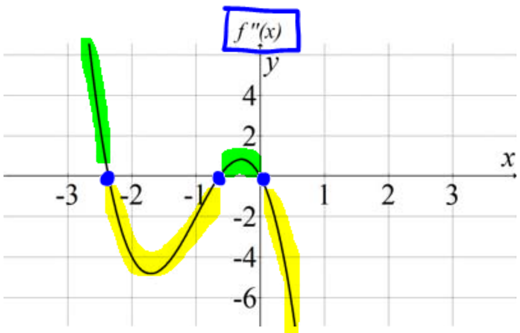
root[f''(x)]

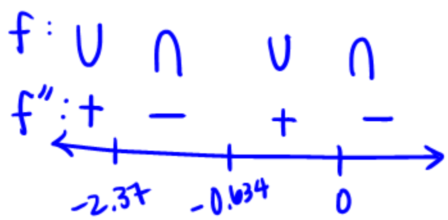
Answer:

(−2.366, 0)

(−0.634, 0)

(0, 0)





Concave Up:

(−∞, −2.366) ∪ (−0.634, 0)

Concave Down:

(−2.366, −0.634) ∪ (0, ∞)

e. Coordinates of any inflection points.

Command:

inflectionpoint[f]

Answer:

(−2.366, −8.2637)

(−0.634, −4.8863)

(0, −5)

17. Let $f(x) = \dfrac{45}{1 + 2e^{-7x}}$. Find Riemann sums with midpoints and 6 subdivisions to approximate the area between the function and the x-axis on the interval [1, 9]. Then approximate the area between the curve and the x-axis using upper sums with 50 rectangles on the interval [-2, 2],

Recall: "Position of rectangle start": 0 corresponds to left endpoints, 0.5 corresponds to midpoints and 1 corresponds to right endpoints.

Enter the function into GGB.

Command:

rectanglesum[f, 1, 9, 6, 0.5]

Answer:

359.9999

Command:

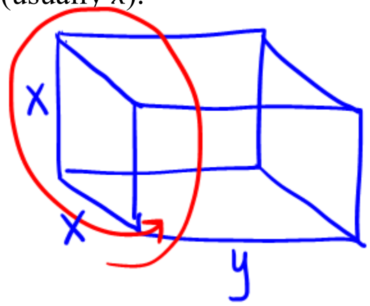
uppersum[f, −2, 2, 50]

Answer:

87.3441

18. Postal regulations state that the girth plus length of a package must be no more than 104 inches if it is to be mailed through the US Postal Service. You are assigned to design a package with a square base that will contain the maximum volume that can be shipped under these requirements. What should be the dimensions of the package? (Note: girth of a package is the perimeter of its base.)

1. Determine the function that describes the situation, and write it in terms of one variable (usually *x*).



$104 = 4x + y \Rightarrow y = -4x + 104$

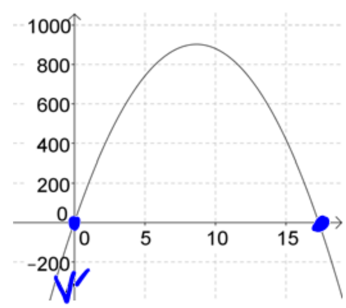
MAX $V = x^2 y$

$V(x) = x^2(-4x + 104)$

$\boxed{V(x) = -4x^3 + 104x^2}$

2. Find its derivative using GGB.
Command:

$V'(x)$



3. Find any critical points.
Command:

root[V'(x)]

Answer:

$(\cancel{0}, 0)$

$(17.3333, 0)$

4. Verify you have a maximum.
Command:

$V''(17.3333)$

Answer:

$-207.9992 < 0$

Abs Max at $17\frac{1}{3}$

5. Dimensions?

$x = 17.3333$ inches

$y = -4x + 104$

$y = -4(17.3333) + 104 = 34.6667$ inches

$\boxed{17\frac{1}{3} \text{ in. by } 17\frac{1}{3} \text{ in by } 34\frac{2}{3} \text{ in}}$

19. Evaluate the following. $\int_{1.3}^{1.5} \frac{6.95x^2}{\sqrt{3.65x - 1.95}}\,dx$

Enter the function into GGB.

Command: Answer:

integral[f, 1.5, 1.3]

1.5335

20. A company estimates that the value of its new production equipment depreciates at the rate of $\frac{dv}{dt} = 10000(t-9)$ $0 \le t \le 9$, where v gives the value of the equipment after t years. Find the total decline in value of the equipment over the first 5 years.

Enter the function into GGB.

a. Setup the integral needed to answer the question.

$$\int_0^5 10000(t-9)\,dt$$

b. Find the total decline in value of the equipment over the first 5 years.

Command: Answer:

integral[f, 0, 5]

-325000

$\boxed{\text{decline by \$325000}}$

21. The temperature in Minneapolis over a 12 hour period can be modeled by the function $C(t) = -0.06t^3 + 0.2t^2 + 3.7t + 5.3$ where t is measured in hours with $t = 0$ corresponding to the temperature at 12 noon. Find the average temperature during the period from noon until 7 p.m. Recall: $\frac{1}{b-a}\int_a^b f(x)dx$

[0, 7]

Enter the function into GGB.

a. Set-up the integral needed to answer the question.

$$\frac{1}{7-0}\int_0^7 C(t)\,dt$$

b. Find the average temperature during the period from noon until 7 p.m.

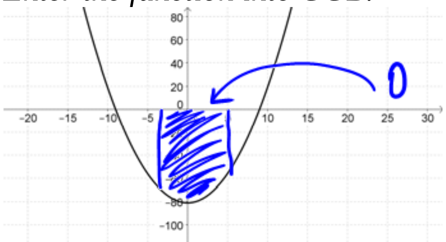
Command: Answer:

1/7 * integral [C, 0, 7]

16.3717°

22. Find the area bounded by the graph of $f(x) = x^2 - 81$, the x-axis and the lines $x = -3$ and $x = 5$. *Recall: The general "formula" for computing the area between two curves is* $\int_a^b (\text{top function - bottom function})dx$. The command is: IntegralBetween[<Function>, <Function>,<Start x-Value>, <End x-Value>]

Enter the function into GGB.



a. Set-up the integral needed to calculate the desired area.

$$\int_{-3}^{5}\left[0-(x^2-81)\right]dx = \int_{3}^{5}(-x^2+81)\,dx$$

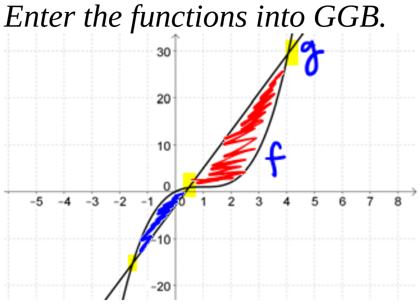
b. Calculate the area.

Command: Answer:

integralbetween[0, f, -3, 5]   597.3333

23. Find the area of the region(s) that is/are completely enclosed by the graphs of $f(x) = (x-1)^3 + 1$ and $g(x) = 8x - 3$.

Enter the functions into GGB.



a. Find the points of intersection.

Command: Answer:

intersect[f, g]

(-1.5341, y)
(0.4827, y)
(4.0514, y)

b. Set-up the integrals needed to calculate the desired area.

$$\int_{-1.5341}^{0.4827}(f-g)\,dx + \int_{0.4827}^{4.0514}(g-f)\,dy$$

c. Calculate the area.

Command: Answer:

integralbetween[f, g, -1.5341, 0.4827] + integral between[g, f, 0.4827, 4.0514]

$= \boxed{35.0501}$

24. Suppose the demand function for a product is *x* thousand units per week and the corresponding wholesale price, in dollars, is $D(x)=\sqrt{174-8x}$. Determine the consumers' surplus if the wholesale market price is set at $8 per unit.

a. Find the quantity demanded.

Command: intersect[D, 8]

Answer: (13.75, 8)

13.75 * 1000 = $\boxed{\text{13,750 units}}$

b. Find the consumers' surplus if the market price for the product is $8 per unit.

Recall: $CS=\int_0^{Q_E} D(x)\,dx-Q_E\cdot P_E$

First apply the formula.

$$CS=\int_0^{13.75}\sqrt{174-8x}\,dx-13.75*8$$

Command: integral[D, 0, 13.75] − 110

Answer: 38.6015 * 1000

$\boxed{=\$38{,}601.50}$

To find the Producers' Surplus, recall: $PS=Q_E\cdot P_E-\int_0^{Q_E} S(x)\,dx$

25. Let $f(x,y)=6x^3-5x^2y+4xy^2+8y^3+3$.

a. Find *f*(1, 1).

Command: f(1,1)

Answer: 16

b. Find the first order partial derivatives.

Commands:

derivative[f, x]

derivative[f, y]

Answer:

$f_x=a(x,y)=18x^2-10xy+4y^2$

$f_y=b(x,y)=-5x^2+8xy+24y^2$

c. Find the second order partial derivatives.

Commands:

derivative[a, x]

derivative[b, y]

derivative[a, y]

derivative[b, x]

Answer:

$f_{xx}=36x-10y$

$f_{yy}=8x+48y$

$\begin{cases} f_{xy}=-10x+8y \\ f_{yx}=-10x+8y \end{cases}$

For the next problem, recall:

$D(x,y) = f_{xx} f_{yy} - (f_{xy})^2$

- If $D(a, b) > 0$ and $f_{xx}(a,b) < 0$, then f has a relative maximum at (a, b).
- If $D(a, b) > 0$ and $f_{xx}(a,b) > 0$, then f has a relative minimum at (a, b).
- If $D(a, b) < 0$, then f has neither a relative maximum nor a relative minimum at (a, b) (i.e., it has a saddle point, which is neither a max nor a min).
- If $D(a, b) = 0$, then this test is inconclusive.

26. Use the second derivative test to find the relative extrema of $f(x, y) = 5x^3 - 2xy + 6y^2$.
Enter the function into GGB.

a. Find the first-order partials.

Commands: Answers:

derivative[f, x] $f_x = a(x,y) = 15x^2 - 2y$

derivative[f, y] $f_y = b(x,y) = -2x + 12y$

b. Find the point of intersection of the equations in part A. These points of intersection are the critical points of the function f.

Command: Answer: $(0,0)$, $(0.0222, 0.0037)$

intersect[a(x,y)=0, b(x,y)=0]

c. Determine $D(x,y) = f_{xx} \cdot f_{yy} - \left(f_{xy}\right)^2$.

Commands: Answers:

derivative[a, x] $f_{xx} = 30x$

derivative[b, y] $f_{yy} = 12$

derivative[a, y] $f_{xy} = -2$

Hence, $D(x,y) = f_{xx} \cdot f_{yy} - \left(f_{xy}\right)^2 = (30x)(12) - (-2)^2 = 360x - 4$

d. Apply the second derivative test to classify each critical point found in step B

$(0,0)$: $D(0,0) = 360(0) - 4 < 0 \Rightarrow$ saddle point at $(0,0)$.

$(0.0222, 0.0037)$: $D = 360(0.0222) - 4 > 0$

$f_{xx} = 30(0.0222) > 0 \Rightarrow$ Rel Min at $(0.0222, 0.0037)$

e. For any maxima point and minima point found in part D, calculate the maxima and minima, respectively.

Command: Answer:

f(0.0222, 0.0037) -0.000027

Formulas to be provided. It will be a link.

$$\frac{f(x+h)-f(x)}{h}=\frac{f(b)-f(a)}{b-a}$$

$C(x)=cx+F$

$R(x)=sx$ or $R(x)=xp$

$P(x)=R(x)-C(x)$

$$\overline{C(x)}=\frac{C(x)}{x}$$

$$E(p)=-\frac{p\cdot f'(p)}{f(p)}$$

$$CS=\int_0^{Q_E} D(x)\,dx-Q_E\cdot P_E$$

$$PS=Q_E\cdot P_E-\int_0^{Q_E} S(x)\,dx$$

$$\frac{1}{b-a}\int_a^b f(x)dx$$

$D(x,y)=f_{xx}f_{yy}-(f_{xy})^2$

- If $D(a, b) > 0$ and $f_{xx}(a,b)<0$, then f has a <u>relative maximum</u> at (a, b).
- If $D(a, b) > 0$ and $f_{xx}(a,b)>0$, then f has a <u>relative minimum</u> at (a, b).
- If $D(a, b) < 0$, then f has neither a relative maximum nor a relative minimum at (a, b) (i.e., it has a saddle point, which is neither a max nor a min).
- If $D(a, b) = 0$, then this test is inconclusive.