Math 1314
Lesson 10
Elasticity of Demand

Suppose you owned a small business and needed to make some decisions about the pricing of your products. It would be helpful to know what effect a small change in price would have on the demand for your product. If a price change will have no real change on demand for the product, it might make good sense to raise the price. However, if a price increase will cause a big drop in demand, then it may not be a good idea to raise prices.

There is a measure of the responsiveness of demand for product or service to a change in its price: **elasticity of demand**. This is defined as

$$\frac{\text{percentage change in demand}}{\text{percentage change in price}}$$

The derivation of the following formula can be found in the online book.

Elasticity of Demand

$E(p) = -\dfrac{p \cdot f'(p)}{f(p)}$, where p is price and $f(p)$ is the demand function, differentiable at $x = p$.

The expression $\dfrac{p \cdot f'(p)}{f(p)}$ is almost always negative, but since we're interested in ***magnitude (size)***, we work with the negative of this ratio to give the elasticity of demand.

Revenues Response To Elasticity

If demand is **elastic** at p, then
- An increase in unit price will cause revenue to decrease.
- A decrease in unit price will cause revenue to increase.

Example: Airfare Demand is said to be **elastic** if $E(p) > 1$.

If demand is **unitary** at p, then
- An increase in unit price will cause the revenue to stay about the same.

This type of elasticity really does not occur. But the best example that can be given to describe it would be: a price increase of 5% will result in a reduction in demand of 5%; a price reduction of 10% will result in an increase in demand of 10%. Demand is said to be **unitary** if $E(p) = 1$.

If demand is **inelastic** at p, then
- An increase in the unit price will cause revenue to increase.
- A decrease in unit price will cause revenue to decrease.

Example: Super Bowl Tickets Demand is said to be **inelastic** if $E(p) < 1$.

2016 Levi's Stadium

Capacity 68500 Attendance 71088

Example 1: If $E(p)=\frac{1}{2}$ when p = \$250, what effect will a 1% increase in price have on demand?

Since $E(p)=\frac{1}{2}$ is: A. >1 B. <1 C. $=1$

Demand is: A. Elastic B. Inelastic C. Unitary

Hence, a 1% increase in price will… cause 0.5% increase in revenue.

Example 2: If $E(p)=\frac{3}{2}$ when p = \$250, what effect will a 1% increase in price have on demand?

Since $E(p)=\frac{3}{2}$ is: A. >1 B. <1 C. $=1$

Demand is: A. Elastic B. Inelastic C. Unitary

Hence, a 1% increase in price will… cause 1.5% decrease in revenue

Example 3: Given $x=f(p)=-2p+15$, determine if demand is elastic, inelastic or unitary when p = \$4

Recall: $E(p)=-\frac{p\cdot f'(p)}{f(p)}$ $f'(p)=-2$

$$E(4)=-\frac{4\cdot(-2)}{-2(4)+15}=\frac{8}{7}>1$$

(p, $f'(4)$, $f(4)$)

Example 4: Suppose the demand function for a product is given by $p = -0.02x + 400$. This function gives the unit price in dollars when x units are demanded.

a. Find the elasticity of demand.

Step 1: Solve the equation for x, so that you have f(p).

$$p = -0.02x + 400$$

$$\frac{p - 400}{-0.02} = \frac{-0.02x}{-0.02}$$

$$x = \frac{p-400}{-0.02} = \frac{p}{-0.02} + \frac{400}{0.02} = -50p + 20000$$

Step 2: Find the derivative of f(p).

$$x(p) = -50p + 20000$$

$$f'(p) = -50$$

Step 3: Apply the formula: $E(p) = -\dfrac{p \cdot f'(p)}{f(p)}$

$$= \frac{-p(-50)}{-50p + 20000}$$

$$= \frac{50p}{-50p + 20000}$$

b. Find $E(\$100)$ and interpret the results.

$$E(100) = \frac{50(100)}{-50(100) + 20000} = 0.\overline{3} < 1$$

Demand is inelastic

c. If the unit price is \$100, will raising the price result in an increase in revenues or a decrease ~~on demand~~? in revenue

increase

d. Find $E(\$300)$ and interpret the results.

$$E(300) = \frac{50(300)}{-50(300) + 20000} = 3 > 1$$

Demand is elastic

e. If the unit price is \$300, will raising the price result in an increase in revenues or a decrease on demand? in revenue