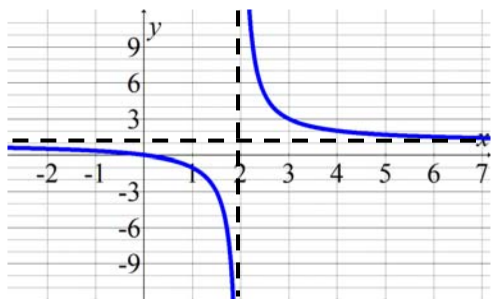
Math 1314
Lesson 13
Analyzing Other Types of Functions

Asymptotes

We will need to identify any vertical or horizontal asymptotes of the graph of a function. A **vertical asymptote** is a vertical line $x = a$ that the graph approaches as values for x get closer and closer to a. A **horizontal asymptote** is a horizontal line $y = b$ that the graph of a function approaches as values for x increase or decrease without bound. The graph of the function shown below has a vertical asymptote at $x = 2$ and a horizontal asymptote at $y = 1$.



If you are given the graph of the function, you can usually just find asymptotes visually by looking at the graph of the function.

When given a function (and not a graph), you can use these rules for finding asymptotes.

Rational functions may have vertical asymptote(s), horizontal asymptote(s), both or neither. To determine asymptotes of rational functions, you can simply apply rules learned in College Algebra or since we can use GGB, simply use the command: asymptote[<function>]

Example 1: Let $f(x) = \frac{x^2 - 4}{x^2 - 3x + 2}$ find any asymptotes.
Command:

Horizontal Asymptote: Vertical Asymptote:

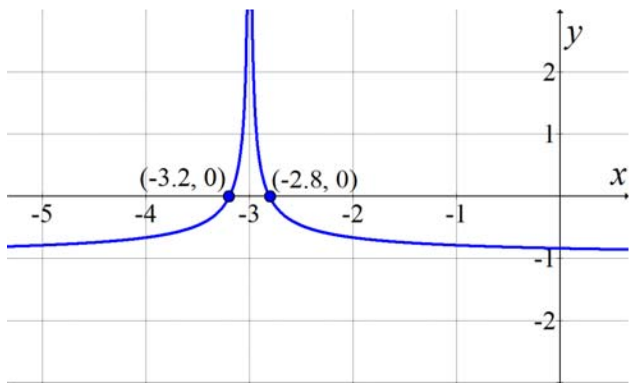
If the function you need to analyze is something other than a polynomial function, you will have some other types of information to find and some analysis techniques will be slightly different.

When working with functions different from polynomials, the critical numbers are defined as follows:

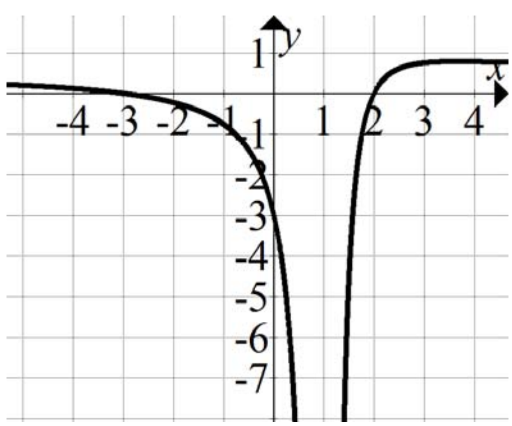
The **critical numbers** of a function are numbers in the domain of the function where $f'(x) = 0$ or where $f'(x)$ is undefined.

You must use caution when the graph of the derivative shows an asymptote, a vertical tangent line or a sharp turn in the graph of the function. If this point is in the domain of the function*, it is also a critical number and must be used in the analysis of the function*.

Example 2: The graph shown below is the derivative of $f(x) = (x+3)^{\frac{1}{3}} - x$. The domain of the function is $(-\infty, \infty)$. Find any critical numbers.



Example 3: The graph shown below is the graph of the derivative of a function. The domain of the original function is $(-\infty, 1) \cup (1, \infty)$. Find any critical numbers.

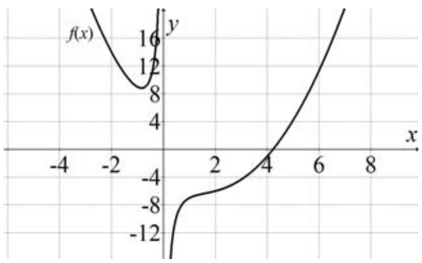


For functions different from polynomials, finding increasing/decreasing intervals, relative extrema and concavity intervals is the same as what we used in analyzing polynomial functions. The commands are slightly different and you must look at its graph for help:

Roots[<Function>, <Start x-Value>, <End x-Value>]
Extremum[<Function>, <Start x-Value>, <End x-Value>]

inflectionpoint[<polynomial>] will NOT WORK FOR FUNCTIONS DIFFERENT FROM POLYNOMIALS. So we'll simply use the line test to determine any points of inflection for other types of functions.

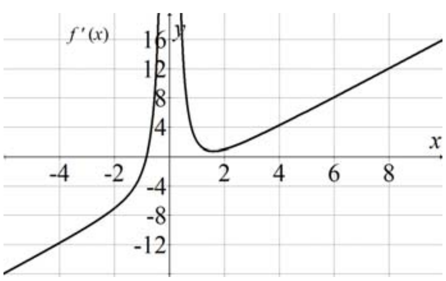
Example 4: Analyze $f(x) = \dfrac{x^3 - 4x^2 - 4}{x}$. *Enter the function in GGB.*



a. Domain

b. Equations of any asymptotes.
Command: Answer:

c. Interval(s) on which the function is increasing; interval(s) on which the function is decreasing.
Command:

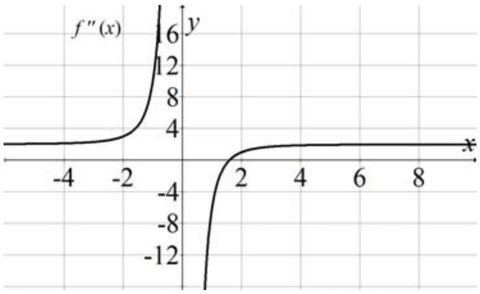


Decreasing: Increasing:

d. Coordinates of any relative extrema.

e. Interval(s) on which the function is concave upward; interval(s) on which the function is concave downward.

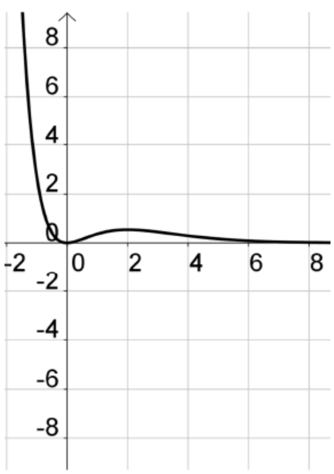
Command:



Concave Up: Concave Down:

f. Coordinates of any inflection points.

Example 5: Analyze $f(x) = x^2 e^{-x}$. *Enter the function in GGB.*



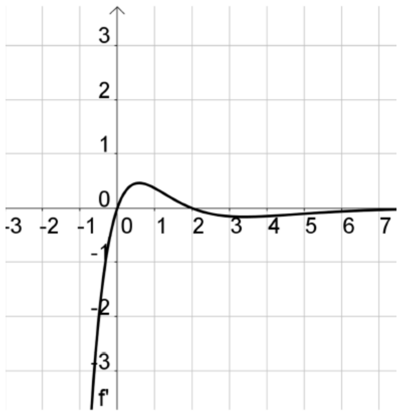
a. Domain.

b. Equations of any asymptotes.

Command: Answer:

c. Interval(s) on which the function is increasing; interval(s) on which the function is decreasing.

Command:

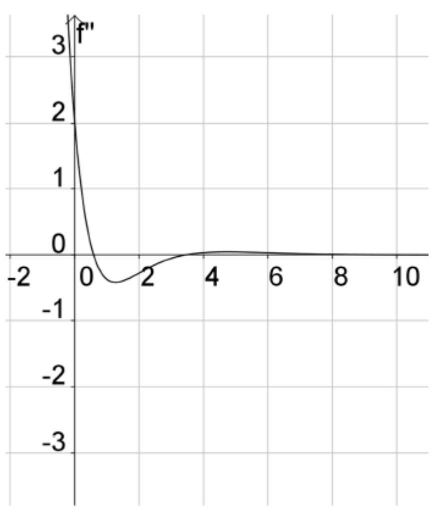


Decreasing: Increasing:

d. Coordinates of any relative extrema.

e. Interval(s) on which the function is concave upward; interval(s) on which the function is concave downward.

Command:



Concave Up: Concave Down:

f. Coordinates of any inflection points.