**Math 1314**
**Lesson 14**
**Optimization**

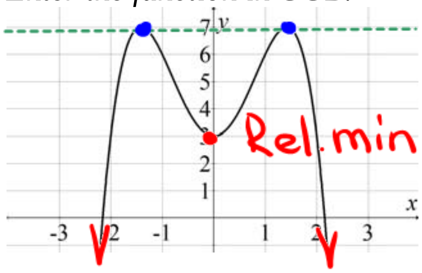
When you optimize something, you make it as large as possible or as small as possible under certain stated conditions. Business owners wish to make revenues and profits as large as possible, while keeping costs as small as possible. There are many other applications as well. In this unit, we'll start by looking at optimization generally; then we'll move on to some applications.

**Absolute Extrema**

**Definition:** If $f(x) \le f(c)$ for all $x$ in the domain of $f$, then $f(c)$ is called the **absolute maximum value** of $f$. If $f(x) \ge f(c)$ for all $x$ in the domain of $f$, then $f(c)$ is called the **absolute minimum value** of $f$.

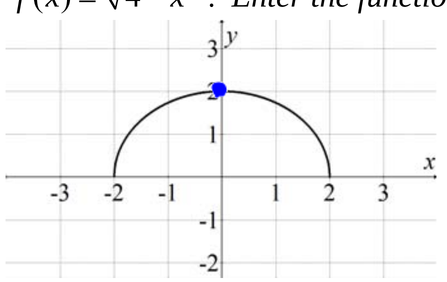
So the *y* value of the point that has the biggest *y* value in the interval of interest is the absolute maximum value. The *y* value of the point that has the smallest *y* value in the interval of interest is the absolute minimum value. We can find absolute extrema by looking at a graph of a function.

Example 1: Find the absolute maximum and absolute minimum values of $f(x) = 3 + 4x^2 - x^4$.
*Enter the function in GGB.*



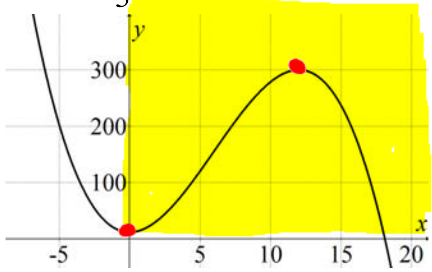

Absolute Maximum Value y = 7 Absolute Minimum Value DNE

Example 2: Find the absolute maximum and absolute minimum values of the function $f(x) = \sqrt{4 - x^2}$. *Enter the function in GGB.*




Absolute Maximum Value y = 2 Absolute Minimum Value y = 0

Example 3: The height of a rocket, $t$ seconds after launch, is given by the function $h(t) = -\frac{1}{3}t^3 + 6t^2 + 12$, where $h(t)$ is given in feet. *Enter the function in GGB.*




Domain : $[0, \infty)$

a. Find the time $t$ when the rocket reaches its maximum height.

Command: Answer:

extremum[h]

$\overset{t}{(0}, \overset{h}{12)}$ & $\overset{t}{(12}, \overset{h}{300)}$ 12 sec

abs. max

b. What is the maximum height of the rocket?

$\boxed{h = 300 \text{ ft}}$

Example 4: The cost of manufacturing $x$ fishing rods is given by $C(x) = 500 + 7x + \frac{1}{10000}x^2$.

The price of each rod is a function of the number manufactured and is given by $p = 10 - 0.0004x$. If all fishing rods manufactured can be sold, what is the number of rods that yields a maximum profit? *Produce the profit function and enter it into GGB.*

*Recall:* $P(x) = R(x) - C(x)$, *where* $R(x) = px$

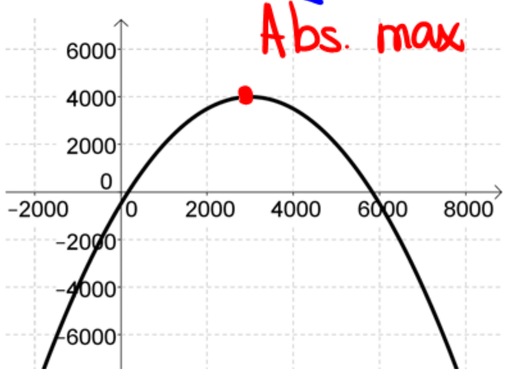
$R(x) = px = (10 - 0.0004x)x = 10x - 0.0004x^2$

$P(x) = R(x) - C(x) = 10x - 0.0004x^2 - (500 + 7x + \frac{1}{10000}x^2)$

Command: Answer:

extremum[P]

Abs. max




(3000, 4000)

\# of rods max profit

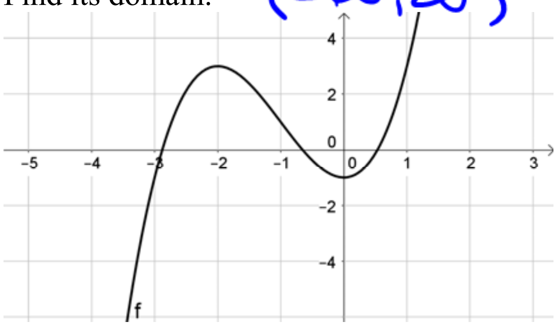
3000 fishing rods yields max. profit

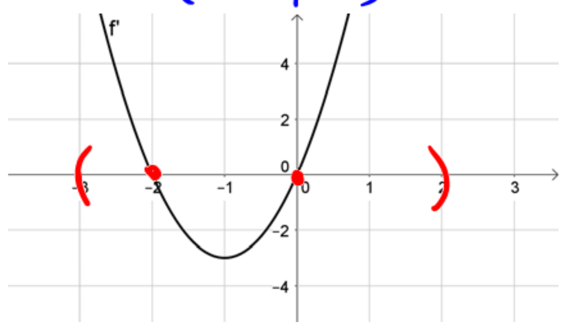
## Finding the Absolute Extrema of *f* on a Closed Interval

1. Find the critical points of *f* that lie in $(a, b)$. *Recall: Critical points are x-values in the domain of the function, where the derivative is equal to 0 or undefined.*
2. Compute the value of the function at every critical point found in step 1 and also compute $f(a)$ and $f(b)$.
3. The absolute maximum value will be the largest value found in step 2, and the absolute minimum value will be the smallest value found in step 2.

Example 5: Find the absolute extrema of $f(x) = x^3 + 3x^2 - 1$ over [-3, 2].
*Enter the function in GGB, find its domain then find the function's derivative.*

Find its domain. $(-\infty, \infty)$ $(-\infty, \infty)$




Where is the derivative equal to zero in (-3, 2)?

Command: root [f']

Answer: (0,0) & (-2,0)

Both are in [-3,2]

Is the derivative undefined in (-3, 2)?

NO!

Now compute the value of the function at every critical point found, and also at the end points of the given interval.

Command:

Answer:

$f(0) = -1$ $f(-3) = -1$

$f(-2) = 3$ $f(2) = 19$

Absolute Minimum Value:

$y = -1$ at $x = 0$ & $x = -3$

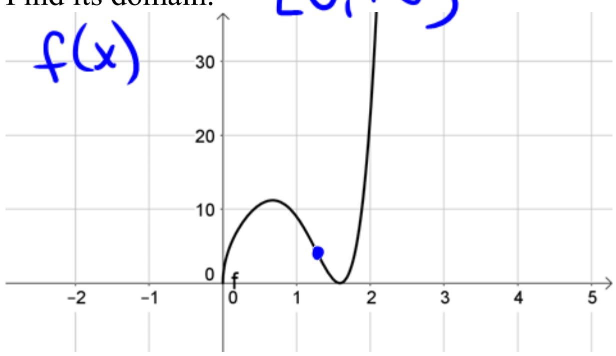
Absolute Maximum Value:

$y = 19$ at $x = 2$

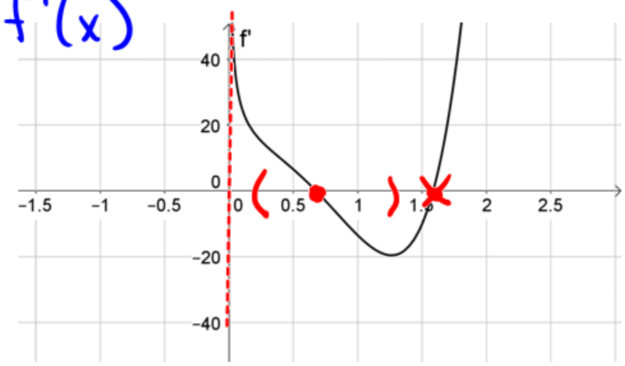
Example 6: Find the absolute extrema of the function $f(x) = \sqrt{x}\left(x^3 - 4\right)^2$ on [0.25, 1.25]. *Enter the function in GGB, find its domain then find the function's derivative.*

Find its domain. $[0, \infty)$

$f(x)$




$f'(x)$




f' is undefined at $x=0$
BUT $x=0$ is NOT included in [0.25, 1.25]

Where is the derivative equal to zero in (0.25, 1.25)?

Command: roots[f', 0.25, 1.25]

Answer: (0.6751, 0) ~~(1.587, 0)~~ NOT in [.25, 1.25]

Is the derivative undefined in ~~(0.5, 1)~~? (0.25, 1.25)

NO!

Now compute the value of the function at every critical point found, and also at the end points of the given interval.

Command:

Answer:

$f(0.6751) = 11.201$ $f(1.25) = 4.684$
$f(0.25) = 7.937$

Absolute Minimum Value:

Absolute Maximum Value:

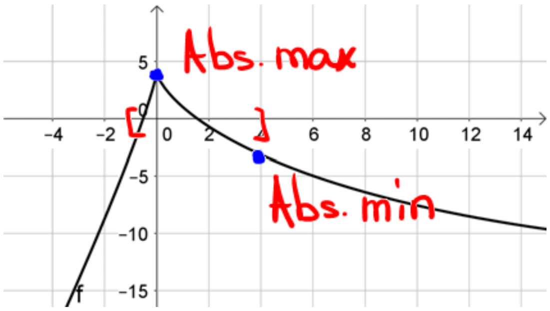


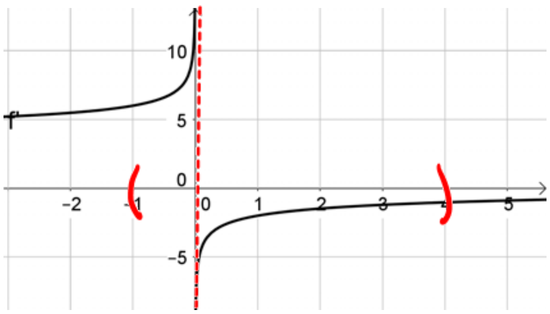
$y = 4.684$ at $x = 1.25$

$y = 11.201$ at $x = 0.6751$

Example 7: Find the absolute extremum of the function $f(x) = 2x - 5x^{4/5} + 4$ on [-1, 4]. *Enter the function in GGB, find its domain then find the function's derivative.*

Find its domain. $(-\infty, \infty)$




Where is the derivative equal to zero in (-1, 4)?
Command: Answer:

NO zeroes on (-1, 4)

Is the derivative undefined in (-1, 4)?

YES! at $x = 0$ ⇐ critical point

Now compute the value of the function at every critical point found, and also at the end points of the given interval.
Command: Answer:

$f(0) = 4$ $f(4) = -3.157$
$f(-1) = -3$

Absolute Minimum Value: $y = -3.157$ at $x = 4$

Absolute Maximum Value: $y = 4$ at $x = 0$