Math 1314
Lesson 19
Numerical Integration

For more complicated functions, we will use GeoGebra to find the definite integral.

The command is: integral[<Function>, <Start x-Value>, <End x-Value>]

The "<Start x-Value>" is the lower limit of integration and the "<End x-Value>" is the upper limit of integration.

Begin each problem by entering the function into GGB.

Example 1: Evaluate: $\int_{-3}^{1} \left(xe^{x} \right) dx$

Command: Answer:

integral [f, -3, 1]   0.1991

Example 2: Evaluate $\int_{3}^{4} \frac{18.6}{1+1.2e^{-4x}} dx$

Command: Answer:

integral [g, 3, 4]   18.6

Example 3: Evaluate $\int_{1}^{2} \frac{x^2}{3x^3+6} dx$

Command: Answer:

integral [h, 1, 2]   0.1338

We can use integration to solve a variety of problems.

Suppose we are given the rate at which a worker can produce items along an assembly line. This is a derivative – a rate of change. When we find the antiderivative, we are finding a function that gives us the total number of items that can be produced. So in this instance, the area under the curve will give total production.

Example 4: A study of worker productivity shows that the rate at which a typical worker can produce widgets on an assembly line can be expressed as $N(t) = -3t^2 + 12t + 15$ where t gives the number of hours after a worker's shift has begun.

a. Set up the integral that represents the number of widgets a worker can produce during the last hour of a five hour shift.

$$\int_4^5 (-3t^2 + 12t + 15)\, dt$$
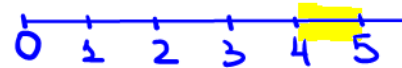
b. Determine the number of widgets a worker can produce during the last hour of a five hour shift.

Command: Answer:

$\text{integral}[f, 4, 5]$ 8 widgets

The velocity function is the derivative of the position function. When we find the antiderivative of the velocity function, we have the position function, and the area under the curve will give the total distance traveled.

Example 5: Suppose you are driving a car and that your velocity can be approximated by $v(t) = 2t\sqrt{25 - t^2}$, where t is measured in seconds and v is measured in feet per second.

a. Set up the integral that represents the distance traveled by the car in the first 5 seconds.

$$\int_0^5 2t\sqrt{25 - t^2}\, dt$$

b. How far will you travel in the 5 seconds from $t = 0$ to $t = 5$?

Command: Answer:

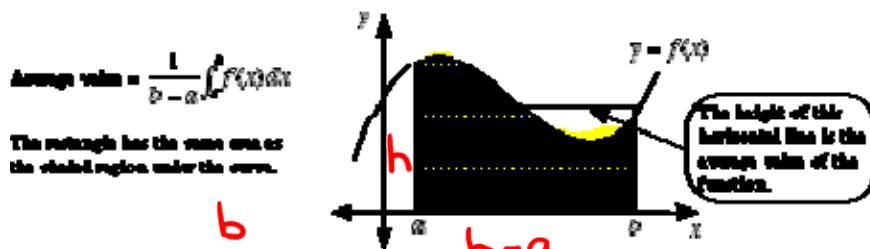$\text{integral}[f, 0, 5]$ 83.3333 feet

So, when you "go backwards" from a function that gives a rate of change, you get a function that gives the total quantity for that function. Then the area under the curve is the total quantity for that function between two given numbers, a and b.

The Average Value of a Function

We can use the definite integral to find the average value of a function.

Suppose f is an integrable function on the interval $[a, b]$. Then the average value of f over the interval is $\boxed{\dfrac{1}{b-a}\int_a^b f(x)dx}$. This is what average value represents:

Average value $= \dfrac{1}{b-a}\int_a^b f(x)dx$

The rectangle has the same area as the shaded region under the curve.

The height of this horizontal line is the average value of the function.

$\dfrac{h(b-a)}{b-a} = \dfrac{\int_a^b f(x)dx}{b-a} \cdot \dfrac{1}{b-a}$

$h = \dfrac{1}{b-a}\int_a^b f(x)dx$

Example 6: Find the average value of $f(x) = x^2 - 3x + 5$ on $[2, 5]$.

a. Set up the integral that represents the average value of the function over the specified interval.

Recall: $\dfrac{1}{b-a}\int_a^b f(x)dx$

$\dfrac{1}{5-2}\int_2^5 (x^2 - 3x + 5)dx$

b. Determine the average value.

Command:

$\frac{1}{3} * \text{integral}[f, 2, 5]$

Answer:

7.5

Example 7: The sales of ABC Company in the first t years of its operation is approximated by the function $S(t) = t\sqrt{0.4t^2 + 4}$ where $S(t)$ is measured in millions of dollars. What were the company's average annual sales over its first five years of operation?

a. Set up the integral that represents the average value of the function over the specified interval.

Recall: $\dfrac{1}{b-a}\int_a^b f(x)dx$

$\dfrac{1}{5-0}\int_0^5 \left(t\sqrt{0.4t^2 + 4}\right)dt$

b. Determine the average value.

Command:

$\frac{1}{5} * \text{integral}[f, 0, 5]$

Answer:

$7.3972 millions

Lesson 19 – Numerical Integration

3