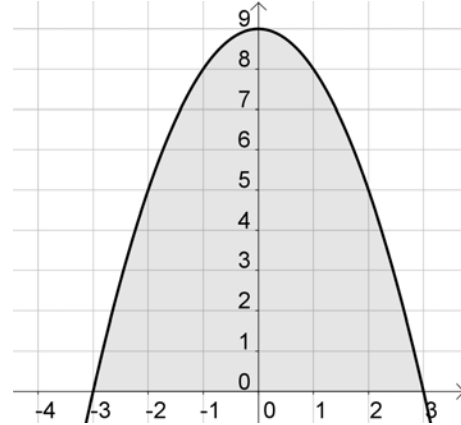# Math 1314
Lesson 20
Area Between Two Curves

The general "formula" for computing the area between two curves is

$\int_a^b (\text{top function} - \text{bottom function})dx$.

Command: integralbetween[<function>, <function>, <start x-value>, <end x-value>]

Example 1: The graph of $f(x) = -x^2 + 9$ is given below. Find the area of the shaded region.

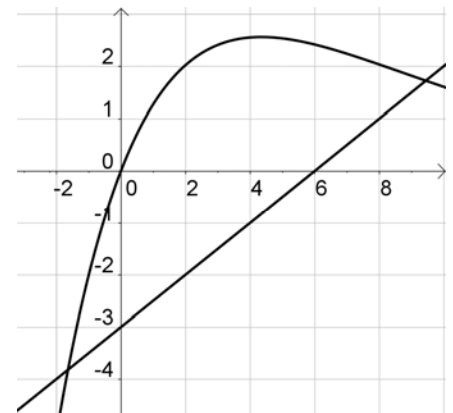a. Set-up the integral needed to calculate the desired area.

b. Calculate the area.

Example 2: Find the area between $f(x) = 1.6xe^{-0.23x}$ and $g(x) = 0.5x - 3$ on the interval $-0.07 \leq x \leq 6.46$.
Enter the functions into GGB.

a. Set-up the integral needed to calculate the desired area.
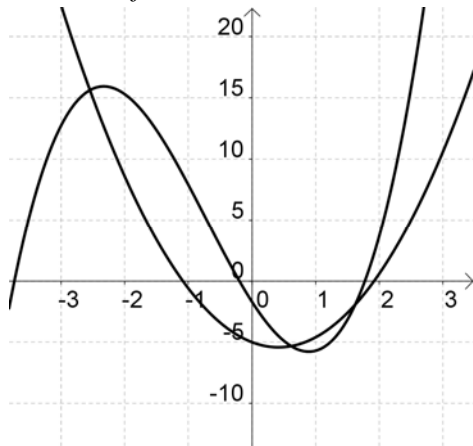

b. Calculate the area.

Sometimes the limits of integration are not given. In this case, we'll need to graph the functions and find the point(s) of intersection of the functions. In GGB we'll use either command:

intersect[<object>, <object>] or intersect[<function>, <function>, <start x-value>, <end x-value>]

Example 3: Find the area completely enclosed by the graphs of the functions
$f(x) = 1.3x^3 + 2.8x^2 - 8.1x - 1.7$ and $g(x) = 2.4x^2 - 2x - 5$.
Enter the functions into GGB.

a. Find the points of intersection.
Command: Answer:

b. Set-up the integrals needed to calculate the desired area.

c. Calculate the area.
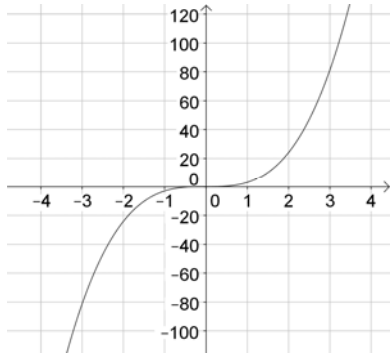Command: Answer:

Example 4: Find the area of the region that is completely enclosed by the graph of $f(x) = 3x^3$ from $x = -3$ to $x = 3$.

Enter the function into GGB.

a. Set-up the integral(s) needed to calculate the desired area.

b. Calculate the area.
Command: Answer:

Now let's look at some applications.

Two advertising agencies are competing for a major client. The rate of change of the client's revenues using Agency A's ad campaign is approximated by $f(x)$ below. The rate of change of the client's revenues using Agency B's ad campaign is approximated by $g(x)$ below. In both cases, x represents the amount spent on advertising in thousands of dollars. In each case, total revenue is the area under the curve given in thousands of dollars.
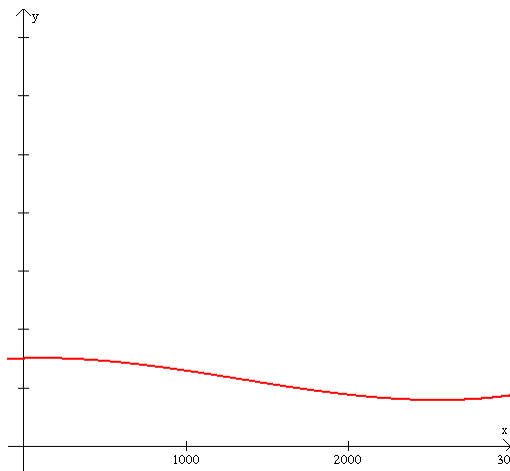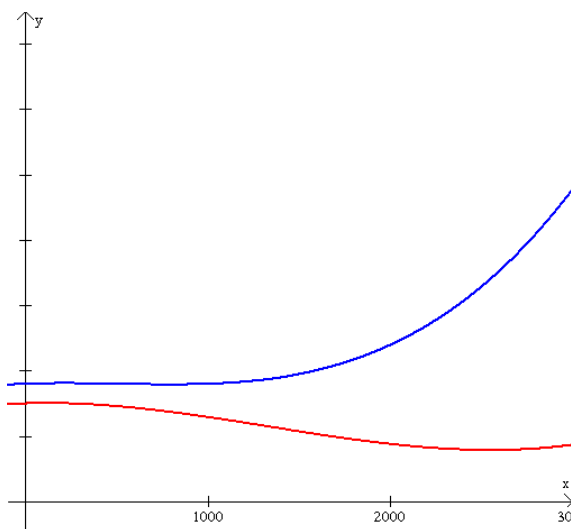
Agency A   Agency B
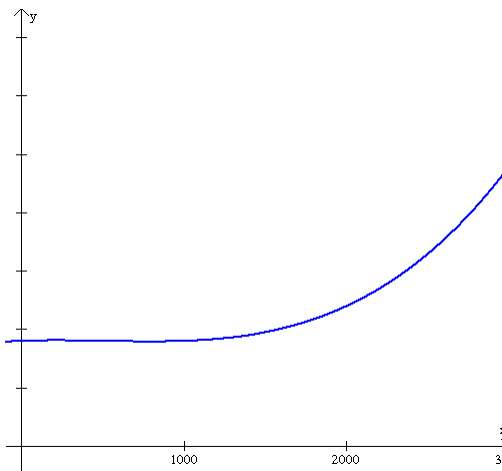
Now let's put the two graph on the same grid. The graph below shows the relationship between the two revenue functions. We see that one function is above the other. ***The area between the two functions represents the projected additional revenue that would be realized by using Agency B's ad campaign.***

This is an example of the kinds of problems you will be able to solve with the techniques you learn in this lesson.

Example 5: Without any effort to curb population growth, a government estimates that its population will grow at the rate of $f(t) = 60e^{0.02t}$ thousand people per year. However, they believe that an education program will alter the growth rate to $g(t) = -t^2 + 60$ thousand people per year over the next 5 years. How many fewer people would there be in the country if the education program is implemented and is successful?

Enter the functions into GGB.
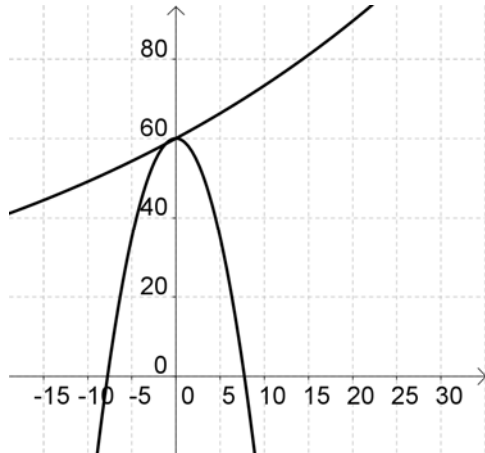
a. Find the point of intersection.
Command: Answer:

b. Set-up the integral needed to calculate the desired area.

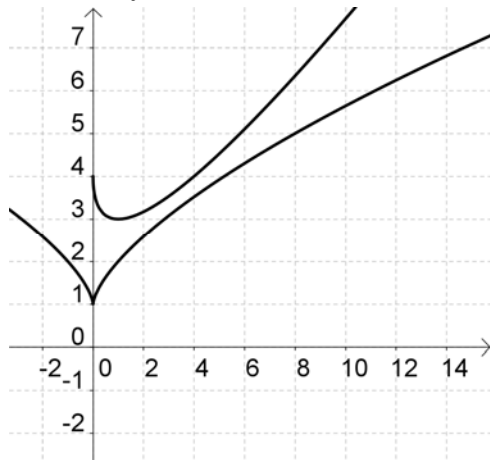c. Calculate the area.
Command: Answer:

Try this one: The management of a hotel chain expects its profits to grow at the rate of $f(t) = 1 + t^{\frac{2}{3}}$ million dollars per year t years from now. If they renovate some of their existing hotels and acquire some new ones, their profits would grow at the rate of $g(t) = t - 2\sqrt{t} + 4$ million dollars per year. Find the additional profits the company could expect over the next ten years if they proceed with their renovation and acquisition plans.

Enter the functions into GGB.

a. Set-up the integrals needed to calculate the desired area.

b. Calculate the area.
Command: Answer: