

## Lesson 22 Functions of Several Variables

So far, we have looked at functions of a single variable. In this section, we will consider functions of more than one variable.

### Functions of Two Variables

Definition: A real valued function of two variables,  $f$ , consists of a set  $A$  of ordered pairs of real numbers  $(x, y)$  called the domain of the function, and a rule that associates with each ordered pair in the domain of  $f$  one and only one real number, denoted by  $z = f(x, y)$ .

$$(5, 3) \neq (3, 5)$$

You will need to learn two skills using functions of several variables: Evaluating at a given point and determining the domain.

Example 1: Suppose  $f(x, y) = xe^{2x} - 5xy^2 + \ln(xy)$  Compute  $f(-1, -3)$ .  
Enter the function as shown in GGB.

$$\ln(x * y)$$

Command:

$$f(-1, -3)$$

Answer:

$$45.9633$$

Example 2: The volume of a cylindrical tank with radius  $r$  and height  $h$  is given by the formula  $f(r, h) = \pi r^2 h$ . Find the volume of a tank with radius 6 feet and height 20 feet.  
Enter the function as shown in GGB.

$$\pi = \text{pi}$$

Command:

$$f(6, 20)$$

Answer:

$$2261.9467$$

Example 3: The monthly payment that amortizes a loan of  $A$  dollars in  $t$  years when the interest rate is  $r$  per year is given by

$$P = f(A, r, t) = \frac{Ar}{\left(12 \left(1 - \left(1 + \frac{1}{12}r\right)^{-12t}\right)\right)}$$

Find the monthly payment for a mortgage of \$250,000 that will be amortized over 25 years with an interest rate of 4.5% per year.

Enter the function as shown in GGB.

$$4.5\% \rightarrow 0.045$$

Command:

$$f(250000, 0.045, 25)$$

Answer:

$$\text{\$ } 1,389.58$$

Example 4: Find the domain of the function:  $f(x, y) = 2x^2 + 3y^2$

$$D: \{(x, y) \mid x, y \in \mathbb{R}\}$$

↑ such that
 ↑ are elements
 ↑ real numbers


Example 5: Find the domain of the function:  $f(x, y) = \frac{3x}{2x-5y}$

$$2x - 5y \neq 0$$

$$2x \neq 5y$$

$$x \neq \frac{5}{2}y$$

$$D: \{(x, y) \mid x \neq \frac{5}{2}y, x, y \in \mathbb{R}\}$$

Recall  
 $f(x) = \frac{1}{x}$   
 $x \neq 0$   
  
 $D: (-\infty, 0) \cup (0, \infty)$


Example 6: Find the domain of the function:  $f(x, y) = \sqrt{16x - y}$

$$16x - y \geq 0$$

$$16x \geq y$$

$$y \leq 16x$$

$$D: \{(x, y) \mid x, y \in \mathbb{R}, y \leq 16x\}$$

Recall  
 $f(x) = \sqrt{x}$   
 $x \geq 0$   
  
 $D: [0, \infty)$