Math 1314
Lesson 7
Derivatives at a Point, Numerical Derivatives and Applications of the Derivative

Numerical Derivatives

Example 1: Find the numerical derivative of $C(x) = 0.000003x^3 - 0.04x^2 + 200x + 70000$ when x = 2500.

Command: C'(2500)

Answer: 56.25

Example 2: Find the numerical derivative of $f(x) = x^{\frac{3}{2}} - e^{2x} + \ln(x)$ when x = 4.

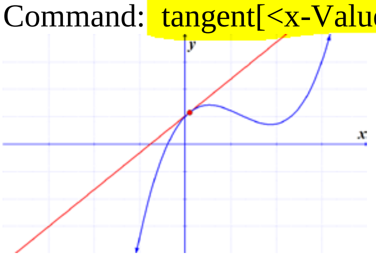
Enter the function into GGB.

Command: f'(4)

Answer: -5958.666

The graph below is the graph of a tangent line to a function at a specific point. Recall that the derivative is a formula for finding the slope of a tangent line to a function at a specific point. Hence, we can find the equation of the tangent line to a function at a specific point.

Command: tangent[<x-Value>,<Function>]



Example 3: Find the slope of the tangent line when x = -1 if $f(x) = \dfrac{x\sqrt{x+2}}{(2x+3)^2}$.

Enter the function into GGB.

Command: f'(-1)

Answer: 4.5

Example 4: Write an equation of the line that is tangent to $f(x) = 1.6x^3 + 6.39x - 2.81$ at (3, 59.56). *Enter the function into GGB.*

Command: tangent[3, f]

Answer: y = 49.59x - 89.21

In other cases, we may want to find all values of x for which the tangent line to the graph of f is horizontal. Since the slope of any horizontal line is 0, we'll want to find the derivative, set it equal to zero and solve the resulting equation for x.

Example 5: Find all x-values on the graph of $f(x)=5-3x+2x^2$ where the tangent line is horizontal. => slope = 0!

$f'(x) = -3+4x = 0$

$4x = 3$

$x = 3/4$

We can also determine values of x for which the derivative is equal to a specified number. Set the derivative equal to the given number and solve for x either algebraically or by graphing.

Example 6: Find all values of x for which $f'(x)=3$: $f(x)=\frac{1}{3}x^3+\frac{5}{2}x^2-7x+3$

$f'(x) = \cancel{\frac{1}{3}} \cdot \cancel{3}x^2 + \frac{5}{\cancel{2}} \cancel{2}x - 7$

$f'(x) = x^2+5x-7 = 3$

$x^2+5x-10=0$

Command:

root [x^2 +5x -10]

Answer:

(-6.5311, 0)

(1.5311, 0)

Many of our problems will ask for the rate at which something is changing at a specific number. Other times, the problem will ask for a function value or an average rate of change (as we saw in the previous lesson). From there we'll apply the appropriate method.

Example 7: The model $N(t)=34.4(1+0.32125t)^{0.15}$ gives the number of people in the US who are between the ages of 45 and 55. Note, $N(t)$ is given in millions and $t = 0$ corresponds to the beginning of 1995. *Enter the function into GGB.*

a. How large is this segment of the population projected to be at the beginning of 2011? => t=16

Command:

N(16)

Answer:

45.1631 million

b. How fast will this segment of the population be growing at the beginning of 2011? = t = 16

Command:

N'(16)

Answer:

0.3545 million/year

Example 8: A study conducted for a specific company showed that the number of lawn chairs assembled by the average worker t hours after starting work at 6 a.m. is given by $N(t) = -t^3 + 7t^2 + 18t$.

a. At what rate will the average worker be assembling lawn chairs at 9 a.m.? => 3 hours later

Command: $N'(3)$

Answer: 33 chairs / hour

b. How many lawn chairs will the average worker have assembled by 12 p.m.? => 6 hours later

Command: $N(6)$

Answer: 144 chairs

c. What is the average rate at which the lawn chairs are assembled from 7 a.m. to 11 a.m.

$$\frac{N(t+h) - N(t)}{h} = \left[\frac{N(5) - N(1)}{4}\right]$$

$[1, 5]$

$h = 5 - 1 = 4$

Command: $(N(5) - N(1))/4$

Answer: 29 chairs / hour

Example 9: The height of a rocket can be modeled by the function $h(t) = -16t^2 + 48t + 6$ where $h(t)$ gives the height in feet at time t given in seconds.

a. What is the height after 2 seconds?

Command: $h(2)$

Answer: 38 feet

b. At what rate is the height changing when $t = 1$?

Command: $h'(1)$

Answer: 16 feet / sec.

c. When will the rocket hit the ground? $h(t) = 0$

Command: root[h]

Answer: ~~(-0.1202, 0)~~

(3.1202, 0)

3.1202 sec

Sometimes we need to find the derivative of the derivative. Since the derivative is a function, this is something we can readily do. The derivative of the derivative is called the second derivative, and is denoted $f''(x)$. To find the second derivative, we will apply whatever rule is appropriate given the first derivative.

Example 10: Find the second derivative: $f(x) = 4x^5 - x^2 - 7x + 5$

$f'(x) = 20x^4 - 2x - 7$

$\boxed{f''(x) = 80x^3 - 2}$

Example 11: Find the value of the second derivative when $x = 5$ if $f(x) = \dfrac{x^2 \ln x}{(x^2+3)^{\frac{1}{3}}}$.

Enter the function into GGB.

Command: $f''(5)$

Answer: 0.8297

An application of the second derivative would be acceleration, as it's the rate at which velocity is changing.