Math 1314
Lesson 9: Marginal Analysis

Marginal Cost

Suppose a business owner is operating a plant that manufactures a certain product at a known level. Sometimes the business owner will want to know how much it costs to produce one more unit of this product. The cost of producing this additional item is called the marginal cost.

Example 1: Suppose the total cost in dollars per week by ABC Corporation for producing its best-selling product is given by $C(x) = 10{,}000 + 3000x - 0.4x^2$. Find the actual cost of producing the 101^{st} item.

First of all, if you calculate $C(101)$, this is the actual cost of producing 101 items. Similarly, $C(100)$ is the actual cost of producing 100 items.

So the cost of producing the 101^{st} item can be found by computing the average rate of change, that is, by computing $\dfrac{C(x+h)-C(x)}{(x+h)-x} = \dfrac{C(101)-C(100)}{101-100}$ where $x = 100$ and $h = 1$.

$= \$2919.6$

This will give us the actual cost of producing the 101^{st} item. However, it is often inconvenient to use. **For this reason, marginal cost is usually approximated by the instantaneous rate of change of the total cost function evaluated at the specific point of interest. That is to say, we'll find the derivative and substitute in our point of interest.**

Example 2: Suppose the total cost in dollars per week by ABC Corporation for producing its best-selling product is given by $C(x) = 10{,}000 + 3000x - 0.4x^2$. Find $C'(100)$ and interpret the results.

$C'(100) = \$2920$

Note that the answers for examples 1 and 2 are very close. This shows you why we can work with the derivative of the cost function rather than the average rate of change. The derivative will be much easier for us to work with. So, we'll define the **marginal cost function** as the derivative of the total cost function.

You will find that by a marginal function, we mean the derivative of the function. So, the marginal cost function is the derivative of the cost function, the marginal revenue function is the derivative of the revenue function, etc.

Example 3: A company produces noise-canceling headphones. Management of the company has determined that the total daily cost of producing x headsets can be modeled by the function $C(x) = 0.0001x^3 - 0.03x^2 + 135x + 15{,}000$.

a. Find the marginal cost function.

$$C'(x) = 0.0003x^2 - 0.06x + 135$$

b. Use the marginal cost function to approximate the actual cost of producing the 25[th] headset.

$$C'(24) = \$133.73 \qquad \lim_{h \to 0} \frac{C(\overset{x+h}{25}) - C(\overset{x}{24})}{\underset{h}{25-24}} = C'(24)$$

Average Cost and Marginal Average Cost

Suppose $C(x)$ is the total cost function for producing x units of a certain product. If we divide this function by the number of units produced, x, we get the average cost function. We denote this function by $\overline{C}(x)$. Then we can express the average cost function as $\overline{C}(x) = \frac{C(x)}{x}$. The derivative of the average cost function is called the **marginal average cost**.

We'll use the marginal average cost function solely to determine if the average cost function is increasing or if it is decreasing.

*If $\overline{C}'(a) > 0$, then the average cost function is increasing when $x = a$.

*If $\overline{C}'(a) < 0$, then the average cost function is decreasing when $x = a$.

Example 4: A company produces office furniture. Its management estimates that the total annual cost for producing x of its top selling executive desks is given by the function $C(x) = 400x + 500{,}000$.

a. Find the average cost function?

$$\overline{C}(x) = \frac{C(x)}{x} = \frac{400x + 500000}{x} = 400 + \frac{500000}{x}$$

b. Find the average cost of producing 3000 desks.

$\bar{C}(3000) = \$566.67$

c. Find the marginal average cost function.
Use GGB to find the derivative:

$$\bar{C}'(x) = -\frac{500000}{x^2}$$

d. Find the $\overline{C'(2587)}$

$\bar{C}'(2587) = -0.747 < 0$

the average cost is decreasing when 2587 desks are produced.

average cost function

e. What happens to $\bar{C}(x)$ when x is very large?

$$\lim_{x \to \infty} \bar{C}(x) = \lim_{x \to \infty} \left(\underbrace{400}_{400} + \underbrace{\frac{500000}{x}}_{0} \right) = \$400$$

$$\lim_{x \to \infty} (f(x) + g(x)) = \lim_{x \to \infty} f(x) + \lim_{x \to \infty} g(x)$$

$$\lim_{x \to \infty} \frac{500000}{x} = 0$$

deg(500000) = 0
deg(x) = 1

deg(N) < deg(D)

Marginal Revenue

We are often interested in revenue functions, as well. The basic formula for a revenue function is given by $R(x) = px$ where x is the number of units sold and p is the price per unit. Often p is given in terms of a demand function in terms of x, which we can then substitute into $R(x)$. The derivative of $R(x)$ is called the **marginal revenue function**. We will use the derivative of the revenue function as an approximation of the actual revenues realized on the sale of the additional item.

Example 5: The table below gives pricing information about a company's best-selling product. Use the data to:

list1 <

x	200	400	600	800	1000
Price when x items are demanded	525	505	491	484	475

a. Create a list. Then find the linear demand function.

1

Command: fitpoly[list1, 1]

Answer: $f(x) = -0.0605x + 532.3$

b. Find the revenue function, then find the marginal revenue function.

$R(x) = px$

$R(x) = (-0.0605x + 532.3)x$

$R(x) = -0.0605x^2 + 532.3x$ $\quad R'(x) = -0.121x + 532.3$

c. Use the marginal revenue function to approximate the actual revenue realized on the sale of the 1101st item.

$R'(1100) = \$399.2$

Marginal Profit

The profit function can be expressed as $P(x) = R(x) - C(x)$, where $R(x)$ is the revenue function and $C(x)$ is the cost function. As before, we will find the marginal function by taking the derivative of the function, so the **marginal profit function** is the derivative of $P(x)$. This will give us a good approximation of the profit realized on the sale of the additional unit of the product.

Example 6: A company estimates that the cost to produce *x* of its products is given by the function $C(x) = 0.000003x^3 - 0.08x^2 + 500x + 250{,}000$ and the demand function is given by $p = 600 - 0.8x$. *Recall:* $P(x) = R(x) - C(x)$ *and* $R(x) = xp$

a. Find the profit function.

$R(x) = xp \qquad R(x) = x(600 - 0.8x)$

$P(x) = R(x) - C(x)$

$P(x) = x(600 - 0.8x) - [0.000003x^3 - 0.08x^2 + 500x + 250000]$

b. Find the marginal profit function.

$P'(x) = -0.000009x^2 - 1.44x + 100$

c. Use the marginal profit function to compute the actual profit realized on the sale of the 53rd unit.

$P'(52) = \$25.09$