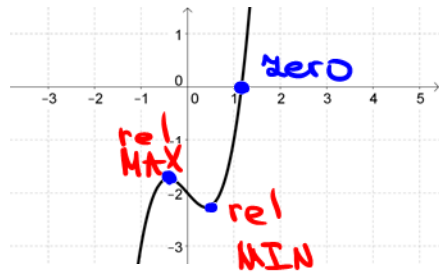
14 Questions

**Math 1314**
**Test 2 Review**
**Lessons 2 – 8**

1. Given $f(x) = 2x^3 - x - 2$.

GGB!




A. Find any zeros of *f*.
Command:

root[f]

Answer:

(1.1654,0)

B. Find any local (relative) extrema of *f*.
Command:

extremum[f]

Answer:

(-0.4082, -1.7278) rel MAX

(0.4082, -2.2722) rel MIN

C. Find $f'(-0.25)$ and $f''(-0.25)$
Command:

f'(-0.25)

f''(-0.25)

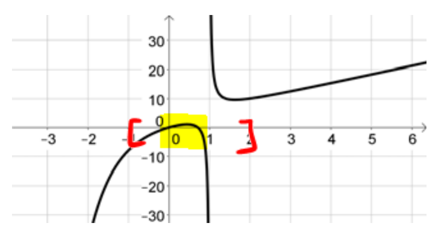
Answer:

-0.625

-3

2. Given $f(x) = \dfrac{2e^{-2x} + 3x^2 - 2}{x - 1}$.

GGB!



A. Find any zeros of *f*.
Command:

roots[f,-1,2]

Answer:

(0,0)

(0.7113,0)

B. Find any extremum of *f*.
Command:

extemum[f,-1,2]

Answer:

(0.4478, 1.0534) Rel MAX

(1.6251, 9.5991) Rel. Min

3. The following table of values gives a company's annual profits in millions of dollars. Rescale the data so that the year 2003 corresponds to $x = 0$. *Create a list of points.*

$x =$ 0 1 2 3 4 5 ... 7

list1 <

| Year | 2003 | 2004 | 2005 | 2006 | 2007 | 2008 |
|---|---|---|---|---|---|---|
| Profits (in millions of dollars) | 31.3 | 32.7 | 31.8 | 33.7 | 35.9 | 36.1 |

2010

linear → deg=1
quadratic → deg=2
cubic → deg=3
quartic → deg=4

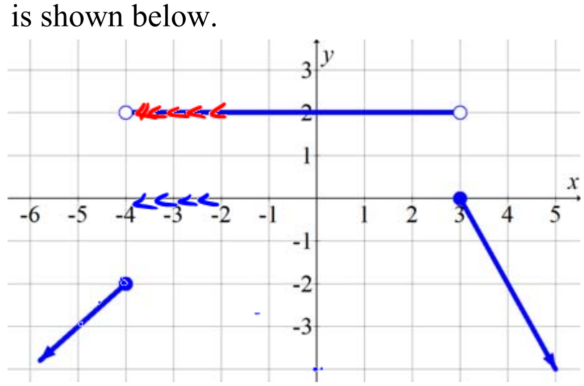
A. Find the cubic (deg=3) regression model for the data.
Command: fitpoly[list 1, 3]
Answer: $f(x) = -0.0566x^3 + 0.531x^2 - 0.3183x + 31.5952$

B. Find the $R^2$ value for the cubic regression model.
Command: rsquare[list 1, f]
Answer: 0.8948

C. Use the cubic regression model to predict the company's profits in 2010.
Command: f(7)
Answer: \$ 36.3286 millions

D. Find the exponential regression model for this data.
Command: fitexp[list 1]
Answer: $g(x) = 31.1057e^{0.03x}$

4. The graph of $f(x)$ is shown below.



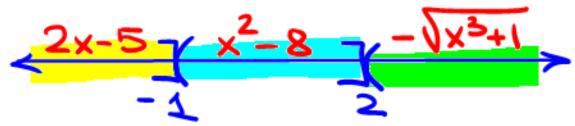
A. $\lim_{x \to -4^-} f(x) = -2$

B. $\lim_{x \to -4^+} f(x) = 2$

C. $\lim_{x \to -4} f(x)$ DNE

Do not agree

5. Suppose $f(x)=\begin{cases} 2x-5, & x\le -1 \\ x^2-8, & -1<x\le 2 \\ -\sqrt{x^3+1}, & x>2 \end{cases}$




Determine, if they exist,

A. $\lim\limits_{x\to -1^-} f(x)$

$= 2(-1)-5$

$= -7$

B. $\lim\limits_{x\to -1^+} f(x)$

$= (-1)^2-8$

$= -7$

C. $\lim\limits_{x\to -1} f(x)$ $= -7$

D. $\lim\limits_{x\to 2^-} f(x)$

$= (2)^2-8$

$= -4$

*E.* $\lim\limits_{x\to 2^+} f(x)$

$= -\sqrt{2^3+1}$

$= -3$

*F.* $\lim\limits_{x\to 2} f(x)$ DNE

6. $\lim\limits_{x\to 2}\left(-3x^2+8\right)$

$= -3(2)^2+8 = \boxed{-4}$

7. $\lim\limits_{x\to 1}\dfrac{\sqrt{4x}}{x-5} = \dfrac{\sqrt{4(1)}}{1-5} = \dfrac{2}{-4}$

$= \boxed{-\frac{1}{2}}$

8. $\lim\limits_{x\to 4}\dfrac{x+3}{2x-8}$

$= \dfrac{4+3}{8-8} = \dfrac{7}{0}$ DNE

9. $\lim\limits_{x\to -5}\dfrac{x^2-25}{x}$

$= \dfrac{(-5)^2-25}{-5} = \dfrac{0}{-5} = \boxed{0}$

10. $\lim\limits_{x\to -2}\dfrac{x^2+5x+6}{x+2} = \dfrac{(-2)^2+5(-2)+6}{-2+2} = \dfrac{0}{0}$

Command:

lim[f,-2]

Answer:

$\boxed{1}$

11. $\lim\limits_{x\to 0}\dfrac{\sqrt{x+9}-3}{x} = \dfrac{\sqrt{0+9}-3}{0} = \dfrac{0}{0}$

Command:

lim[g,0]

Answer:

$\boxed{0.1667}$

***Limits at infinity: Compare the degree of the numerator and the degree of the denominator.***

$x \to \infty$ or $-\infty$

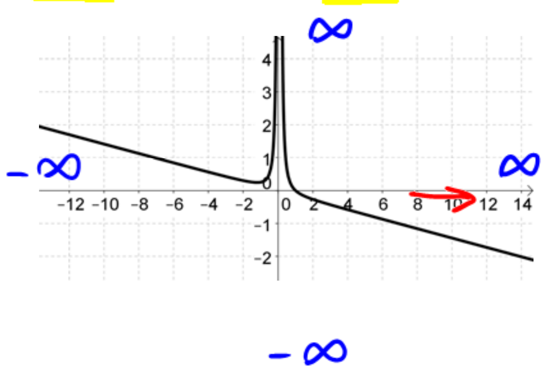
- If the degree of the numerator is smaller than the degree of the denominator, then $\lim\limits_{x\to\infty}\dfrac{f(x)}{g(x)}=0.$
- If the degree of the numerator is the same as the degree of the denominator, then you can find $\lim\limits_{x\to\infty}\dfrac{f(x)}{g(x)}$ by making a fraction from the leading coefficients of the numerator and denominator and then reducing to lowest terms.
- If the degree of the numerator is larger than the degree of the denominator, then it's best to work the problem viewing the graph in GGB. You can then decide if the function approaches $\infty$ or $-\infty$. This limit does not exist, but the $\infty$ or $-\infty$ is more descriptive.

12. $\lim\limits_{x\to\infty}\dfrac{10x^2-x}{3-4x^2} = \dfrac{10}{-4} = \boxed{-\dfrac{5}{2}}$

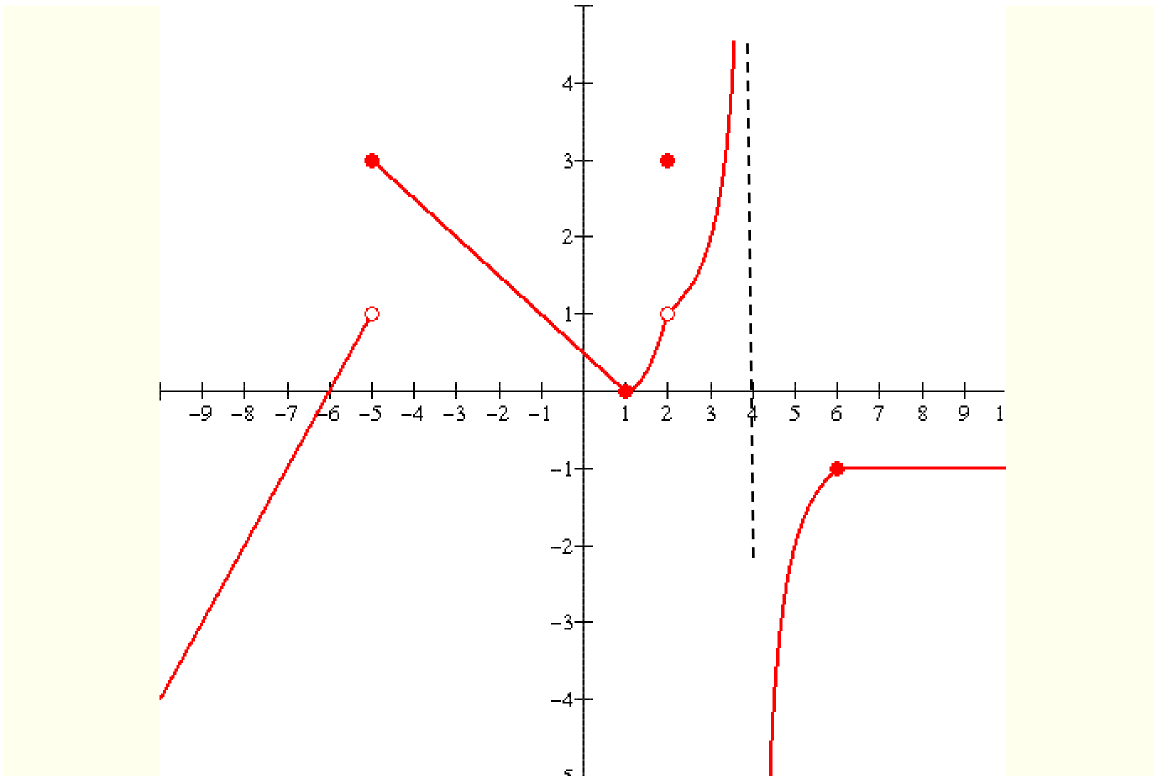
13. $\lim\limits_{x\to-\infty}\dfrac{x^3+5x^2-7x-1}{2+x^2-7x^4} = 0$

14. $\lim\limits_{x\to\infty}\dfrac{x^3-1}{x-7x^2} = -\infty$ $\qquad \lim\limits_{x\to-\infty} = \infty$

*Enter the function into GGB. Look at the graph to determine your answer.*



15. The graph of $f(x)$ is shown below. Which of the following statements is true?




I. $\lim_{x\to 2} f(x)$ exists and is equal to 3. False

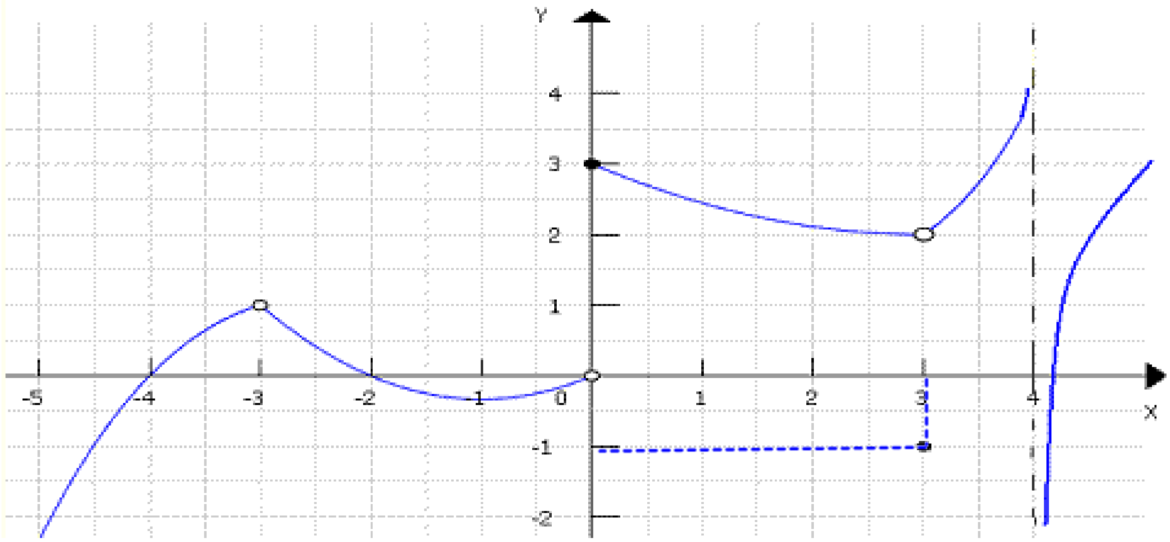
II. $\lim_{x\to -5} f(x)$ exists and is equal to 3. False

III. $\lim_{x\to 6} f(x)$ does not exist. False

IV. $\lim_{x\to 2} f(x)$ does not exist; there is a hole where $x = 2$. False

V. $\lim_{x\to 4} f(x)$ does not exists; there is unbounded behavior as $x$ approaches 4. True

16. The graph of $f(x)$ is shown below. Which of the following statements is true?




I. The function is continuous at $x$ = 3. False

II. The function is discontinuous at $x$ = 3 because $\lim_{x\to 3} f(x)$ does not exist. False

III. The function is discontinuous at $x$ = 3 because $f(3)$ does not exist. False

IV. The function is discontinuous at $x$ = 3 because even though $f(3)$ exists and $\lim_{x\to 3} f(x)$ exists, the two quantities are not equal. True

$f(3) = -1$ $\lim_{x\to 3} f(x) = 2$

14. Find the first and second derivative: $f(x) = 5x^4 - 3x^3 + 8x^2 + 7x - 1$

$f'(x) = 20x^3 - 9x^2 + 16x + 7$

$f''(x) = 60x^2 - 18x + 16$

17. Let $f(x) = \dfrac{3x^4 - 5\sqrt{x}}{\ln(x-1) + e^{2x}}$

A. Find the slope of the tangent line at $x = 3$.

Command: $f'(3)$

Answer: $-0.3603$

B. Write the equation of the tangent line at the given point.

Command: tangent[3, f]

Answer: $y = -0.3603x + 1.6608$

GGB!

18. Find the average rate of change of $f(x) = 0.28x^2 - 0.11x$ on the interval $[1.5, 4]$. *Recall:* $\dfrac{f(x+h) - f(x)}{h}$ = average rate of change/difference quotient

$h = 4 - 1.5$
$= 2.5$

$$\frac{f(4) - f(1.5)}{2.5}$$

Command: (f(4) - f(1.5))/2.5

Answer: 1.43

GGB!

19. The model $N(t) = 12000\,(1 + 0.3559\,t)^{0.18}$ gives the number of bacteria in a culture $t$ hours after an experiment begins. What will be the bacteria population 6 hours after the experiment begins?

Command: N(6)

Answer: 14741 bacteria

20. A country's gross domestic product (GDP) in billions of dollars, $t$ years from now, is projected to be $N(t) = 6\,t^2 + 16\,t + 17$ for $0 \le t \le 5$. What will be the rate of change of the country's GDP 2 years from now?

$N'(t) = 12t + 16$

$N'(2) = 12(2) + 16 = \$40$ billion/year

21. A ball is thrown upwards from the roof of a building at time $t = 0$. The height of the ball in feet is given by $h(t) = -16\,t^2 + 148\,t + 78$, where $t$ is measured in seconds. Find the velocity of the ball after 3 seconds.

$h'(t) = -32t + 148$

$h'(3) = -32(3) + 148 = 52$ ft/sec

22. Suppose a manufacturer has monthly fixed costs of $250,000 and production costs of $24 for each item produced. The item sells for $40. Assume all functions are linear. State the:

A. cost function.

$C(x) = mx + b$

*m = cost/unit; b = fixed costs*

GGB!

$C(x) = 24x + 250000$

B. revenue function.

$R(x) = px$

*p = selling price*

GGB!

$R(x) = 40x$

C. profit function.

$P(x) = R(x) - C(x)$

$P(x) = 40x - (24x + 250000)$

$= 16x - 250000$

D. Find the break-even point. *Recall:* $R(x) = C(x)$

$P(x) = 0$

Command:

intersect[C(x), R(x)]

OR

intersect[24x+250000, 40x]

OR

intersect[16x-250000, 0]

OR root[P(x)]

Answer:

(15625, 625000)

15625 units

$ 625000

23. Cost data and demand data for a company's best-selling product are given in the tables below. *Create two lists.*

| | | | | |
|---|---|---|---|---|
| Quantity produced | 1,000 | 2,000 | 3,000 | 4,000 |
| Total cost | \$13,400 | \$14,200 | \$14,900 | \$15,400 |
| Quantity demanded | 1,000 | 2,000 | 3,000 | 4,000 |
| Price in dollars | \$10.75 | \$10.15 | \$9.85 | \$9.70 |

list1 <

list2 <

A. Find linear regression model for cost.

Command: fitpoly[list1, 1]

Answer: $C(x)$ → $f(x) = 0.67x + 12800$

B. Find the linear regression model for demand. Then find the revenue function.

Command: fitpoly[list2, 1]

Linear Demand Equation: $g(x) = -0.0003x + 10.975$

Revenue Equation: *Recall:* $R(x) = px$

$R(x) = g(x) * x = -0.0003x^2 + 10.975x$

D. Use the linear cost and revenue function to find the number of items that must be sold to break even on that product. Round your answer to the nearest unit.

Command: intersect [f, R]

Answer:

(1299, 13670.04)

(28571, 31942.57)

24. Suppose that a company has determined that the demand equation for its product is $5x+3p-30=0$ where $p$ is the price of the product in dollars when $x$ of the product are demanded ($x$ is given in thousands). The supply equation is given by $52x-30p+45=0$, where $x$ is the number of units that the company will make available in the marketplace at $p$ dollars per unit. Find the equilibrium quantity and price.

Command: Answer:

intersect[5x+3p-30=0, 52x-30p+45=0]

Quantity = 2500 units (2.5, 5.83)

Price = $5.83

25. Let y = 25x - 2650 be a supply equation and y = -6.5x + 1760 be a demand equation. Find the equilibrium point.

Command: Answer:

intersect[25x-2650, -6.5x+1760]

(140, 850)

The following formulas will be provided with Test 2.

**It will be a link.**

$$\frac{f(x+h)-f(x)}{h}=\frac{f(b)-f(a)}{b-a}$$

$C(x)=mx+b=cx+F$

$R(x)=sx$ or $R(x)=xp$

$P(x)=R(x)-C(x)$