Math 1314
Test 3 Review
Material covered is from Lessons 9 – 15

GGB→

1. The total weekly cost of manufacturing x cameras is given by the cost function: $C(x) = -0.0001x^3 + 0.4x^2 + 800x + 3,000$. Use the marginal cost function to approximate the cost of producing the 157th camera.

Command: Answer:

$C'(156) = \$917.50$

2. A company has the given demand function: $p = -0.02x + 600$

A. Find the revenue function.

Recall: $R(x) = px$

$R(x) = (-0.02x + 600)x$

B. Use the marginal revenue function to approximate the revenue realized from the sale of the 234th unit.

Command: Answer:

$R'(233) = \$590.68$

3. A music company produces a variety of electric guitars. The total cost of producing x guitars is given by the function $C(x) = 6100 + 7x - \frac{1}{5}x^2$ where $C(x)$ is given in dollars. Find the average cost of producing 130 guitars.

Recall: $\overline{C(x)} = \frac{C(x)}{x}$

$$\bar{C}(130) = \frac{6100 + 7(130) - \frac{1}{5}(130)^2}{130} = \$27.92$$

Know!

Demand is said to be **elastic** if $E(p) > 1$.
Demand is said to be **unitary** if $E(p) = 1$.
Demand is said to be **inelastic** if $E(p) < 1$.

4. Suppose $E(p) = \frac{1}{4}$ when the price of the item is p. Then the demand is
a. Elastic b. Unitary $\boxed{\text{c. Inelastic}}$

5. Suppose the demand equation of a product is given by $p = -0.04x + 1000$ where the function gives the unit price in dollars when x units are demanded. Compute $E(p)$ when $p = 535$ and interpret the results.

Recall: $E(p) = -\dfrac{p \cdot f'(p)}{f(p)}$

Step 1: Solve the equation for x, so that you have f(p).

$p = -0.04x + 1000$

$0.04x + p = 1000$

$\dfrac{\cancel{0.04}x}{\cancel{0.04}} = \dfrac{-p}{0.04} + \dfrac{1000}{0.04}$

$x = -25p + 25000$

$f(p) = -25p + 25000$

Step 2: Find the derivative of f(p).

$f'(p) = -25$

Step 3: Apply the formula: $E(p) = -\dfrac{p \cdot f'(p)}{f(p)}$

$E(p) = -\dfrac{p(-25)}{-25p + 25000} = \dfrac{25p}{-25p + 25000}$

Now compute E(535)

$E(535) = 1.1505 > 1$ Elastic

6. The number of deer present in a nature preserve can be expressed using the model $N(t) = \dfrac{125}{1+31e^{-0.6t}}$, where $N(t)$ gives the number of deer and t gives the number of months since the initial count of deer was taken.

A. How many deer will be present after 6 months?

Command: $N(6)$

Answer: 68 deer

B. At what rate is the population changing after 6 months?

Command: $N'(6)$

Answer: 18.6214 deer / month

7. At the beginning of an experiment, a researcher has 511 grams of a substance. If the half-life of the substance is 16 days:

A. Identify two points given in the problem.

(0, 511) (16, 255.5)

list1

0	511
16	255.5

B. Find an exponential regression model using the two points in part a and GGB.

Command: fitexp[list1]

Answer: $f(x) = 511e^{-0.0433x}$

C. How many grams of the substance are left after 25 days?

Command: $f(25)$

Answer: 173.0061 grams

D. What is the rate of change after 10 days?

Command: $f'(10)$

Answer: -14.3593 grams/day

decay

8. The graph given below is the *first derivative* of a function, *f*.

A. Find any critical numbers of *f*.

$x=-1$

$x=1$

B. Find any intervals where the *f* is increasing/decreasing and any relative extremum.

f: ↘ ↗ ↘

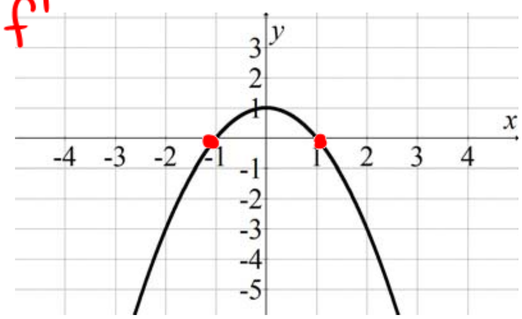
f': $-$ $+$ $-$ (at -1 and 1)

Increasing: $(-1,1)$

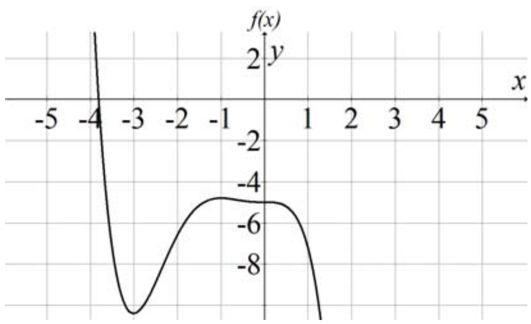
Decreasing: $(-\infty,-1)\cup(1,\infty)$

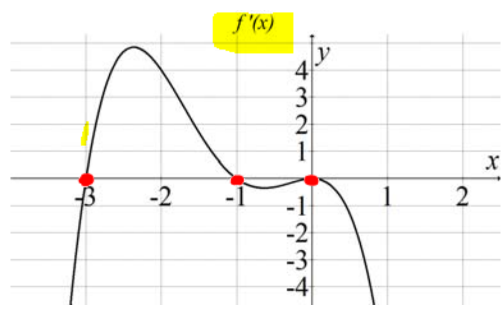
Rel. min at $x=-1$ Rel. max at $x=1$.

f'



9. Let $f(x)=-0.2x^5-x^4-x^3-5$. *Enter the function in GGB.*





A. Find any critical numbers of *f*.

Command: root[f'(x)]

Answer: $x=-3$ $x=-1$ $x=0$

B. Interval(s) on which *f* is increasing; interval(s) on which *f* is decreasing.

f: ↘ ↗ ↘ ↘

f': $-$ $+$ $-$ $-$ (at -3, -1, 0)

Increasing: $(-3,-1)$

Decreasing: $(-\infty,-3)\cup(-1,\infty)$

C. Coordinates of any relative extrema.

Command:

Answer:

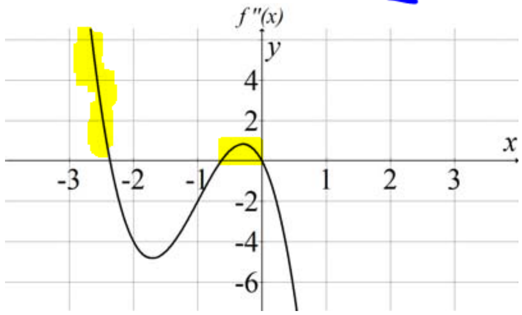
$x=-3$ local min $f(-3)=-10.4$

$x=-1$ local max $f(-1)=-4.8$

D. Interval(s) on which f is concave upward; interval(s) on which f is concave downward.

Commands: Answer:

f'' root[f''(x)] $x = -2.366$ $x = -0.634$ $x = 0$



f: ∪ ∩ ∪ ∩

f'': + − + −

-2.366 -0.634 0

Concave Down: $(-2.366, -0.634) \cup (0, \infty)$

Concave Up: $(-\infty, -2.366) \cup (-.634, 0)$

E. Coordinates of any inflection points.

Command: inflectionpoint[f]

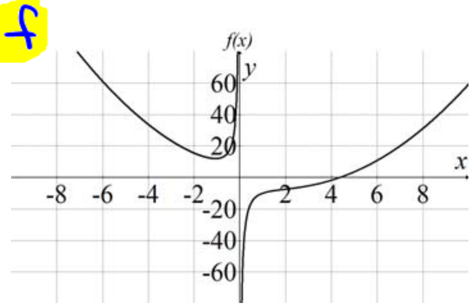
Answer: $(-2.366, -8.2637)$ $(-.634, -4.886$ $(0, -5)$

F. Find any vertical asymptotes and any horizontal asymptotes.

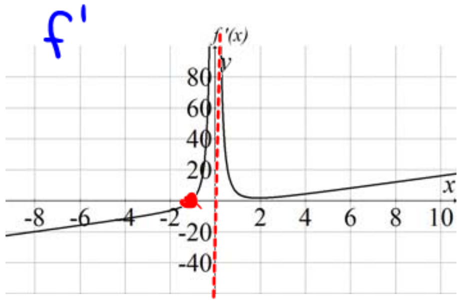
asymptote[f]   none   list{}

10. Let $f(x) = \dfrac{x^3 - 4x^2 - 7}{x}$. *Enter the function in GGB.*

f



f'



A. Find any critical numbers of f.

Command: roots[f']

Answer: $x = -1.0681$ $x = 0$ (asymptote)

B. Interval(s) on which f is increasing; interval(s) on which f is decreasing.

f: ↘ ↗ ↗

f': − + +

-1.0681 0

Increasing: $(-1.0681, 0) \cup (0, \infty)$

Decreasing: $(-\infty, -1.0681)$

C. Coordinates of any relative extrema.

Command: extremum[f, -10, 10]

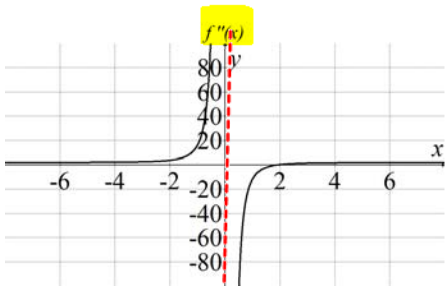
Answer: $(-1.0681, 11.9669)$ Rel. min

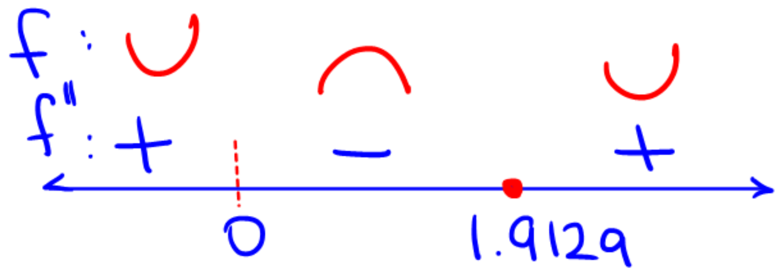
D. Interval(s) on which f is concave upward; interval(s) on which f is concave downward.

Commands: Answer:

f'' roots[f''(x)

$x = 1.9129$ $x=0$





Concave Down: Concave Up:

$(0, 1.9129)$ $(-\infty, 0) \cup (1.9129, \infty)$

E. Coordinates of any inflection points.

$x = 1.9129$

Command: Answer:

$f(1.9129) = -7.6518$

P.O.I.:

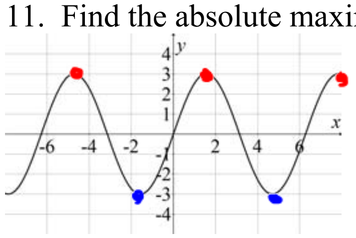
$(1.9129, -7.6518)$

F. Find any vertical asymptotes and any horizontal asymptotes.

Command: Answer:

asymptote[f] $x=0$ (vertical)

11. Find the absolute maximum and absolute minimum of this function.

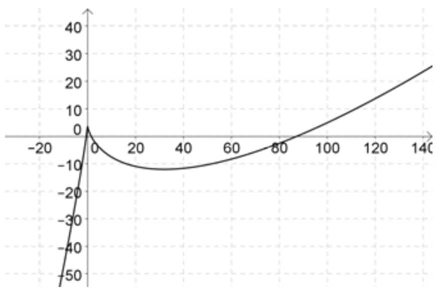


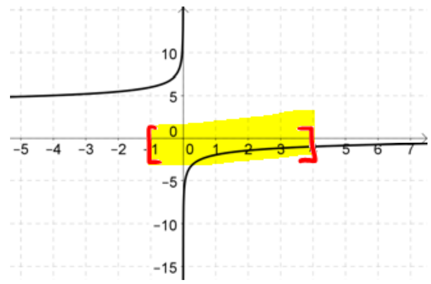
Abs Max: Abs Min:

3 −3

12. Find the absolute extremum of the function $f(x) = 2x - 5x^{4/5} + 4$ on [-1, 4]. *Enter the function in GGB, find its domain then find the function's derivative.*

$f(x)$ Find its domain. $(-\infty, \infty)$ $f'(x)$





Where is the derivative equal to zero in (-1, 4)?

Command: roots[f', 20, 38]

Answer: $x = 32$ NOT in $[-1, 4]$

Is the derivative undefined in (-1, 4)?

Commands:

Answer:

$x = 0 \Leftarrow$ critical point

Now compute the value of the function at every critical point found, and also at the end points of the given interval.

Command:

Answer:

$f(-1) = -3$

$f(4) = -3.1572 \leftarrow$ Abs. min.

$f(0) = 4$ Abs. max

13. The mosquito population is a function of rainfall, and can be approximated by the formula $N(x) = 1000 + 42x^2 - x^3$, where x is the number of inches of rainfall. Note that x is non-negative. What is maximum number of mosquitos?

Command: extremum[f(x)]

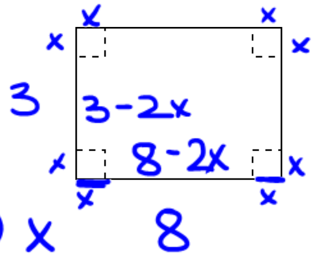
Answer: (0,1000) (28, $\boxed{11976}$)

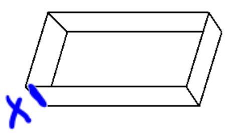
Abs. max

14. If you cut away equal squares from all four corners of a piece of cardboard and fold up the sides, you will make a box with no top. Suppose you start with a piece of cardboard the measures 3 feet by 8 feet. Find the dimensions of the box that will give a maximum volume. What is the maximum volume?

1. Determine the function that describes the situation, and write it in terms of one variable (usually *x*).

$V = lwh$

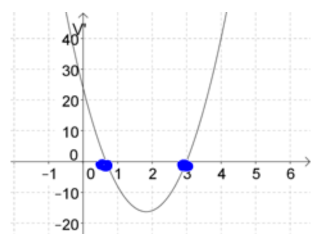




$V = (8-2x)(3-2x)\,x$

2. Find its derivative using GGB.
Command:

$V'(x) = 12x^2 - 44x + 24$



3. Find any critical points.
Command: Answer:

root[V'(x)]

$x = .6667$ ~~$x = 3$~~

4. Verify you have a maximum.
Command: Answer:

$V''(.6667) = -27.9992 < 0$ Abs. max

5. Dimensions?

$x = .6667$ ft ⇐ height

$3 - 2(.6667) = 1.6667$ ft

width

$8 - 2(.6667) = 6.6667$ ft ⇐ length

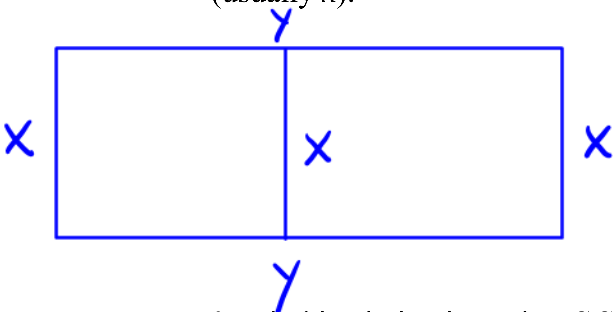
6. Max volume?
Command: Answer:

$V(.6667) =$

$7.4074\ ft^3$

15. A farmer has 420 feet of fencing to enclose 2 adjacent rectangular pig pens sharing a common side. The two adjacent pens have the same dimensions. Find the dimensions of each pen so that the enclosed area is maximized.

1. Determine the function that describes the situation, and write it in terms of one variable (usually *x*).



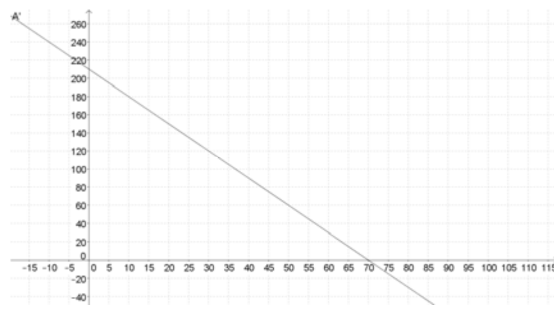
$420 = 3x + 2y$ $\quad A = xy$

Solve for y $\quad$ Max

$y = 210 - \frac{3}{2}x$ $\quad A = x\left(210 - \frac{3}{2}x\right)$

2. Find its derivative using GGB.
Command:

$A'(x)$



3. Find any critical points.
Command: root [A']

Answer: $x = 70$

4. Verify you have a maximum.
Command: $A''(70) = -3 < 0$

Answer: Abs. max

5. Dimensions?

$x = 70$ ft ⇐ width

$y = 210 - \frac{3}{2}(70) = 105$ ft ⇐ length of both

$\frac{105}{2} = 52.5$ ft ⇐ length of each