15 Questions

Math 1314
Test 4 Review
Lesson 16 – Lesson 24

1. Use Riemann sums with midpoints and 6 subdivisions to approximate the area between $f(x) = \dfrac{45}{1 + 2e^{-7x}}$ and the x-axis on the interval [1, 9].

Recall: RectangleSum[<Function>,<Start x-Value>,<End x-Value>,<Number of rectangles>,<Position for rectangle start>]

!

!

"Position of rectangle start": 0 corresponds to left endpoints, 0.5 corresponds to midpoints and 1 corresponds to right endpoints.

Enter the function into GGB.
Command: Answer:

rectanglesum[f, 1, 9, 6, 0.5] 359.999

2. Approximate the area between the curve and the x-axis using upper sums with 50 rectangles on the interval [-2, 2], with $f(x) = x^3 - 7x + 13$

Recall: UpperSum[<Function>,<Start x-Value>,<End x-Value>,<Number of Rectangles>]; LowerSum[<Function>,<Start x-Value>,<End x-Value>,<Number of Rectangles>]

Enter the function into GGB.
Command: Answer:

uppersum[f, -2, 2, 50] 52.6605

3. Find the indefinite integral $\int (2x^3 + 5x^2 - 4x + 3)\,dx$

$$= \frac{\cancel{2}^{\,1}x^4}{\cancel{4}\,2} + \frac{5x^3}{3} - \frac{\cancel{4}^{\,2}x^2}{\cancel{2}} + 3x + C$$

$$= \frac{1}{2}x^4 + \frac{5}{3}x^3 - 2x^2 + 3x + C$$

4. Evaluate the following. $\displaystyle\int_{1.3}^{1.5} \frac{6.95\,x^2}{\sqrt{3.65\,x - 1.95}}\,dx$

Recall: The command is: integral[<Function>, <Start x-Value>, <End x-Value>]
The "<Start x-Value>" is the lower limit of integration and the "<End x-Value>" is the upper limit of integration.

Enter the function into GGB.
Command: integral[f, 1.3, 1.5]

Answer: 1.5335

5. An efficiency study showed that the rate at which the average worker assembles products t hours after starting work can be modeled by $f(t) = -3t^2 + 11t + 25$, where $0 \le t \le 4$. Determine the number of units the average worker can assemble during the third hour that s/he works during a shift. *Enter the function into GGB.*

1 2 3
0 1 2 3

a. Setup the integral needed to answer the question.

$\int_2^3 f(t)\,dt$

b. Answer the question.
Command: integral[f, 2,3]

Answer: 33.5 ≈ 34 units

6. The temperature in Minneapolis over a 12 hour period can be modeled by the function $C(t) = -0.06\,t^3 + 0.2\,t^2 + 3.7\,t + 5.3$ where t is measured in hours with $t = 0$ corresponding to the temperature at 12 noon. Find the average temperature over the first 7 hours.

Recall: $\dfrac{1}{b-a}\displaystyle\int_a^b f(x)dx$

0 1 2 3 4 5 6 7
a b

Enter the function into GGB.
a. Set-up the integral needed to answer the question.

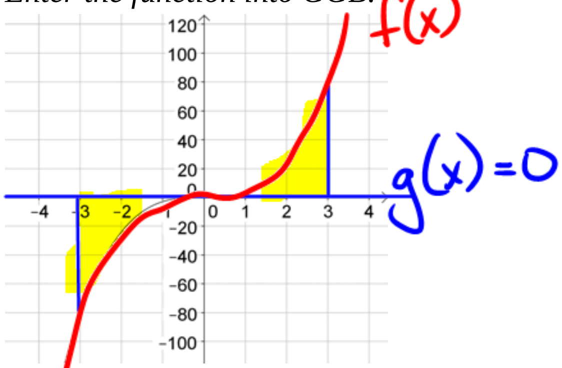
$\dfrac{1}{7-0}\displaystyle\int_0^7 c(t)\,dt$

b. Answer the question.
Command: 1/7*integral[C,0,7]

Answer: 16.3717

7. Find the area bounded by the graph of $f(x) = 3x^3$, the x-axis and the lines $x = -3$ and $x = 3$.
Recall: The general "formula" for computing the area between two curves is
$\int_a^b (\text{top function - bottom function})dx$.
The command is: IntegralBetween[<Function>, <Function>,<Start x-Value>, <End x-Value>]

Enter the function into GGB.



a. Set-up the integral needed to calculate the desired area.

$$\int_{-3}^{0}[0-(3x^3)]dx + \int_{0}^{3}[3x^3-0]dx = \int_{-3}^{0}-3x^3dx + \int_{0}^{3}3x^3dx$$

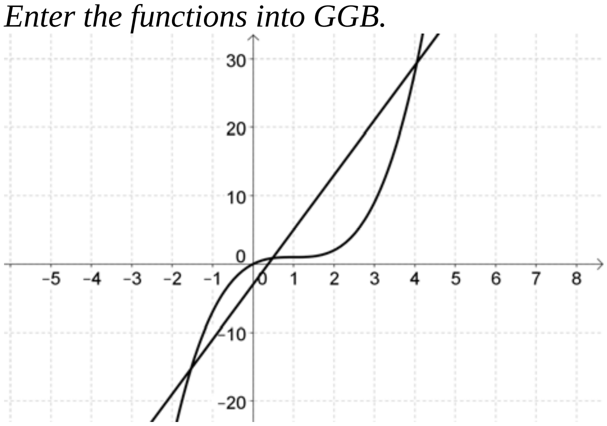
b. Calculate the area.

Command: Answer:

integralbetween[g,f,-3,0] + integralbetween[f,g,0,3]

121.5

8. Find the area of the region(s) that is/are completely enclosed by the graphs of $f(x) = (x-1)^3 + 1$ and $g(x) = 8x - 3$.
Enter the functions into GGB.



a. Find the points of intersection.

Command: Answer:

intersect [f,g]

$x = -1.5341$

$x = 0.4827$

$x = 4.0514$

b. Set-up the integrals needed to calculate the desired area.

$$\int_{-1.5341}^{0.4827} [\overbrace{((x-1)^3+1)}^{f(x)} - \overbrace{(8x-3)}^{g(x)}]dx + \int_{0.4827}^{4.0514} [\overbrace{(8x-3)}^{g(x)} - \overbrace{((x-1)^3+1)}^{f(x)}]dx$$

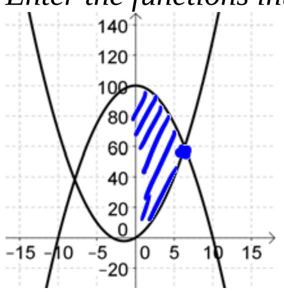
c. Calculate the area.

Command: integralbetween[f,g,-1.5341,.4827] + integralbetween[g,f,.4827,4.0514]

Answer: 35.0501

9. A company is considering a new manufacturing process in one of its plants in an effort to save money. The company estimates that the rate of savings can be modeled by $S(x) = 100 - x^2$ where x is time in years and $S(x)$ is given in thousands of dollars per year. At the same time, the company's operating costs will increase, and the company estimates that the rate at which costs will increase can be modeled by $C(x) = x^2 + 3x$ where x is time in years and $C(x)$ is given in thousands of dollars per year. Find the total net savings that the company should expect to realize.

Enter the functions into GGB.



a. Find the points of intersection.

Command: intersect [C,S]

Answer: ~~x = -7.8607~~

x = 6.3607

b. Set-up the integrals needed to calculate the desired area.

$$\int_{0}^{6.3607} [S(x) - C(x)]\,dx$$

c. Calculate the area.

Command: integralbetween [S,C,0,6.3607]

Answer: 403.8193

$403819.3

10. Suppose the demand function for a product is x thousand units per week and the corresponding wholesale price, in dollars, is $D(x)=\sqrt{174-8x}$. Determine the consumers' surplus if the wholesale market price is set at \$8 per unit.

Recall: $CS=\int_0^{Q_E} D(x)\,dx - Q_E \cdot P_E$

$f(x)=8$

$PS = Q_E \cdot P_E - \int_0^{Q_E} S(x)\,dx$

a. Find the quantity demanded.

Command: intersect[D,f]

Answer: (13.75, 8)

quantity price

b. Find the consumers' surplus if the market price for the product is \$8 per unit.

Recall: $CS=\int_0^{Q_E} D(x)\,dx - Q_E \cdot P_E$

First apply the formula.

$$\int_0^{13.75} \sqrt{174-8x}\,dx - 13.75*8$$

Command: integral[D,0, 13.75] - 13.75*8

Answer: 38.6015

\$ 38601.5

11. The life expectancy of an electrical component has a probability density function that is defined by $f(t)=\frac{1}{2}e^{-\frac{1}{2}t}$ where t is given in months after first use. Find the probability that a randomly selected component will last at most 7 months. *Enter the function into GGB.*

a. Set-up the integral needed to answer the question.

$$\int_0^7 \frac{1}{2}e^{-\frac{1}{2}t}\,dt$$

b. Find the probability that a randomly selected component will last at most 7 months.

Command: integral[f, 0,7]

Answer: .9698

12. Let

$$f(x, y) = \frac{-5.28\,x^2 + 4.37\,x^2\,y + 1.16\,x\,y}{3.94\,y^2 + 2.45\,x\,y^3}$$

Find $f(8, 6)$.

Enter the function into GGB.

Command: f(8,6)

Answer: .319

13. Find the second order partial derivatives of $f(x, y) = 6\,x^3 - 5\,x^2\,y + 4\,x\,y^2 + 8\,y^3 + 3$

Enter the function into GGB.

Commands:

derivative[f, x] $f_x = a(x,y) = 18x^2 - 10xy + 4y^2$

derivative[f, y] $f_y = b(x,y) = -5x^2 + 8xy + 24y^2$

derivative[a, x] $f_{xx} = c(x,y) = 36x - 10y$

derivative[b, y] $f_{yy} = d(x,y) = 8x + 48y$

derivative[a, y] $f_{xy} = e(x,y) = -10x + 8y$

derivative[b, x] $f_{yx} = g(x,y) = -10x + 8y$

Answer:

14. The productivity of a country is given by the Cobb-Douglas function $f(x, y) = 25x^{0.29}y^{0.71}$, where x represents the utilization of labor and y represents the utilization of capital. If the company uses 1,100 units of labor and 775 units of capital, find the marginal productivity of capital.

Commands:

derivative [f,y]

a(1100, 775)

Answer:

$a(x,y) = \frac{71}{4}\,\frac{x^{.29}}{y^{.29}}$

19.6474 units / unit increase in capital

For the next problem, recall:

$D(x,y) = f_{xx} f_{yy} - (f_{xy})^2$

- If $D(a, b) > 0$ and $f_{xx}(a,b) < 0$, then f has a relative maximum at (a, b).
- If $D(a, b) > 0$ and $f_{xx}(a,b) > 0$, then f has a relative minimum at (a, b).
- If $D(a, b) < 0$, then f has neither a relative maximum nor a relative minimum at (a, b) (i.e., it has a saddle point, which is neither a max nor a min).
- If $D(a, b) = 0$, then this test is inconclusive.

15. Find the critical points of $f(x,y) = 5x^3 - 2xy + 6y^2$.
Enter the function into GGB.

a. Find the first-order partials.

Commands:

derivative [f, x]

derivative [f, y]

Answer:

$f_x = a(x,y) = 15x^2 - 2y$

$f_y = b(x,y) = -2x + 12y$

b. Find the point of intersection of the first-order partials. These points of intersection are the critical points of the function *f*.

Command:

$15x^2 - 2y = 0$

$-2x + 12y = 0$

intersect [c, g]

Answer:

c: $15x^2 - 2y = 0$

g: $-x + 6y = 0$

$\boxed{(0,0) \quad (0.0222, 0.003)}$

For the next problem, recall:

$D(x,y) = f_{xx} f_{yy} - (f_{xy})^2$

- If $D(a, b) > 0$ and $f_{xx}(a,b) < 0$, then f has a relative maximum at (a, b).
- If $D(a, b) > 0$ and $f_{xx}(a,b) > 0$, then f has a relative minimum at (a, b).
- If $D(a, b) < 0$, then f has neither a relative maximum nor a relative minimum at (a, b) (i.e., it has a saddle point, which is neither a max nor a min).
- If $D(a, b) = 0$, then this test is inconclusive.

16. Suppose

$$f_{xx} = -30x\,,\ f_{yy} = -6\,,\ f_{xy} = f_{yx} = 4$$

and the critical points for function f are

$$A = (-0.2869, -0.1913) \text{ and } B = (0.4647, 0.3098)$$

Find the value for D for each critical point and then classify the critical point using the second derivative test.

a. Determine $D(x,y) = f_{xx} \cdot f_{yy} - \left(f_{xy}\right)^2$.

$D(x,y) = (-30x)(-6) - (4)^2 = 180x - 16$

b. Apply the second derivative test to classify each critical point.

$\boxed{A}$ $D(-0.2869) = -67.642 < 0$ saddle point

$\boxed{B}$ $D(0.4647) = 67.646 > 0$

$f_{xx} = -30(x) = -30(0.4647) < 0$

Rel. max at (.4647, .3098)

For the next problem, recall:

$D(x, y) = f_{xx} f_{yy} - (f_{xy})^2$

- If $D(a, b) > 0$ and $f_{xx}(a,b) < 0$, then f has a relative maximum at (a, b).
- If $D(a, b) > 0$ and $f_{xx}(a,b) > 0$, then f has a relative minimum at (a, b).
- If $D(a, b) < 0$, then f has neither a relative maximum nor a relative minimum at (a, b) (i.e., it has a saddle point, which is neither a max nor a min).
- If $D(a, b) = 0$, then this test is inconclusive.

17. Suppose that $f(x, y) = -5x^3 + 7xy - 8y^2$, (0.2042 , 0.0893) is a critical point,

$f_{xx}\,|_{(0.2042\,,\,0.0893)} = -6.1250$ < 0 Rel. max

and

$D\,(0.2042\,,\,0.0893) = 49$. > 0

$f(.2042, 0.0893) = 0.0213$

Which of these statements describes the graph of f at (0.2042 , 0.0893) ?

a. f has a relative minimum value at $f\,(0.2042\,,\,0.0893) = 0.0213$.
b. f has a saddle point at $f\,(0.2042\,,\,0.0893) = -0.1064$.
c. f has a relative maximum value at $f\,(0.2042\,,\,0.0893) = -0.1064$.
d. f has a relative maximum value at $f\,(0.2042\,,\,0.0893) = 0.0213$.
e. f has a relative minimum value at $f\,(0.2042\,,\,0.0893) = -0.1064$.
f. f has a saddle point at $f\,(0.2042\,,\,0.0893) = 0.0213$.

Formulas to be provided. It will be a link.

$$CS = \int_0^{Q_E} D(x)\,dx - Q_E \cdot P_E$$

$$PS = Q_E \cdot P_E - \int_0^{Q_E} S(x)\,dx$$

$$\frac{1}{b-a}\int_a^b f(x)dx$$

$$D(x, y) = f_{xx} f_{yy} - (f_{xy})^2$$

- If $D(a, b) > 0$ and $f_{xx}(a,b) < 0$, then f has a <u>relative maximum</u> at (a, b).
- If $D(a, b) > 0$ and $f_{xx}(a,b) > 0$, then f has a <u>relative minimum</u> at (a, b).
- If $D(a, b) < 0$, then f has neither a relative maximum nor a relative minimum at (a, b) (i.e., it has a saddle point, which is neither a max nor a min).
- If $D(a, b) = 0$, then this test is inconclusive.