MATH 2312 Algebra Review Part 2

Asymptotes of Functions

The line x = a is a vertical asymptote of the graph of a function f if f(x) increases or decreases without bound as x approaches a.

Examples:

Given
$$f(x) = \frac{1}{x}$$
, the line $x = 0$ (y-axis) is its vertical asymptote.

Given $f(x) = \frac{x^2}{(x+1)^2}$, the line x = -1 is its vertical asymptote.



Holes: The graph of the function has a hole if there is a common factor of the numerator and denominator.

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Finding Vertical Asymptotes and Holes Algebraically

- 1. Factor the numerator and denominator as much as possible.
- 2. Look at each factor in the denominator.
 - If a factor cancels with a factor in the numerator, then there is a hole where that factor equals zero.
 - If a factor does not cancel, then there is a vertical asymptote where that factor equals zero.

Example: Find the vertical asymptotes for $f(x) = \frac{5x+2}{(x+2)(x-3)}$.

NO holes

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Example: Find all vertical asymptotes for $f(x) = \frac{2x}{1+2\sin x}$ in the interval $[0, 2\pi)$.

$$1 + 2 \sin x = 0$$

$$2 \sin x = -1$$

$$\sin x = -\frac{1}{2}$$

$$x = \frac{11}{6} \qquad x = \frac{111}{6}$$

Horizontal Asymptotes

The line y = b is a **horizontal asymptote** of the graph of a function f if f(x) approaches b as x increases (goes to infinity) or decreases (goes to negative infinity) without bound.

Examples:

Given
$$f(x) = \frac{1}{x}$$
, the line $y = 0$ (x-axis) is its horizontal asymptote.

Given $f(x) = \frac{x^2}{(x+1)^2}$, the line y = 1 is its horizontal asymptote.



Horizontal asymptotes really have to do with what happens to the *y*-values as *x* becomes very large or very small. If the *y*-values approach a particular number at the far left and far right ends of the graph, then the function has a horizontal asymptote.



b is the leading coefficient of the denominator.

0 or 1

Note: A rational function may have several vertical asymptotes, but only at most one horizontal asymptote. In addition, a graph cannot cross a vertical asymptote, but may cross a horizontal asymptote.

Example: Find the horizontal asymptote, if there is one, of:



For those examples above that do have a horizontal asymptote, determine whether the graph of the function crosses it.



$f(x) = x^3 - 5x^2 + 1$

We have discussed horizontal asymptotes of rational functions but many other types of functions have horizontal asymptotes. For example, inverse trigonometric functions, exponential functions.

Note: Polynomial functions do NOT have asymptotes (neither vertical nor horizontal).

Note: A function may have at most two horizontal asymptotes.

Example:



Example: Given the graph below, which of the following can be this function?

