Math 1330
Algebra Review Part 3

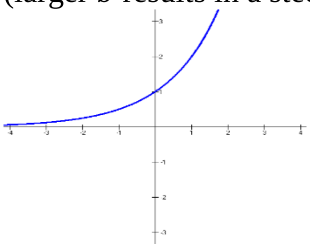
Exponential Functions

Functions whose equations contain a variable in the exponent are called **exponential functions.**

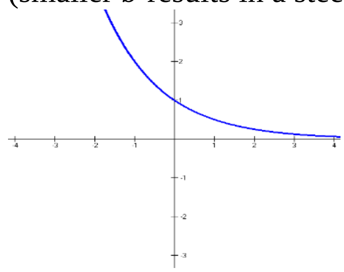
The **exponential function f with base b** is defined by $f(x) = b^x$ $(b > 0 \text{ } and \text{ } b \neq 1)$ and x is any real number.

If $b = e$ (*the natural base*, $e \approx 2.7183$), then we have $f(x) = e^x$.

If $b > 1$, the graph of $f(x) = b^x$ looks like
(larger b results in a steeper graph):



If $0 < b < 1$, the graph of $f(x) = b^x$ looks like
(smaller b results in a steeper graph):



Domain: $(-\infty, \infty)$
Range: $(0, \infty)$
Key Point: $(0,1)$
Both graphs have a horizontal asymptote of y = 0 (the x-axis).

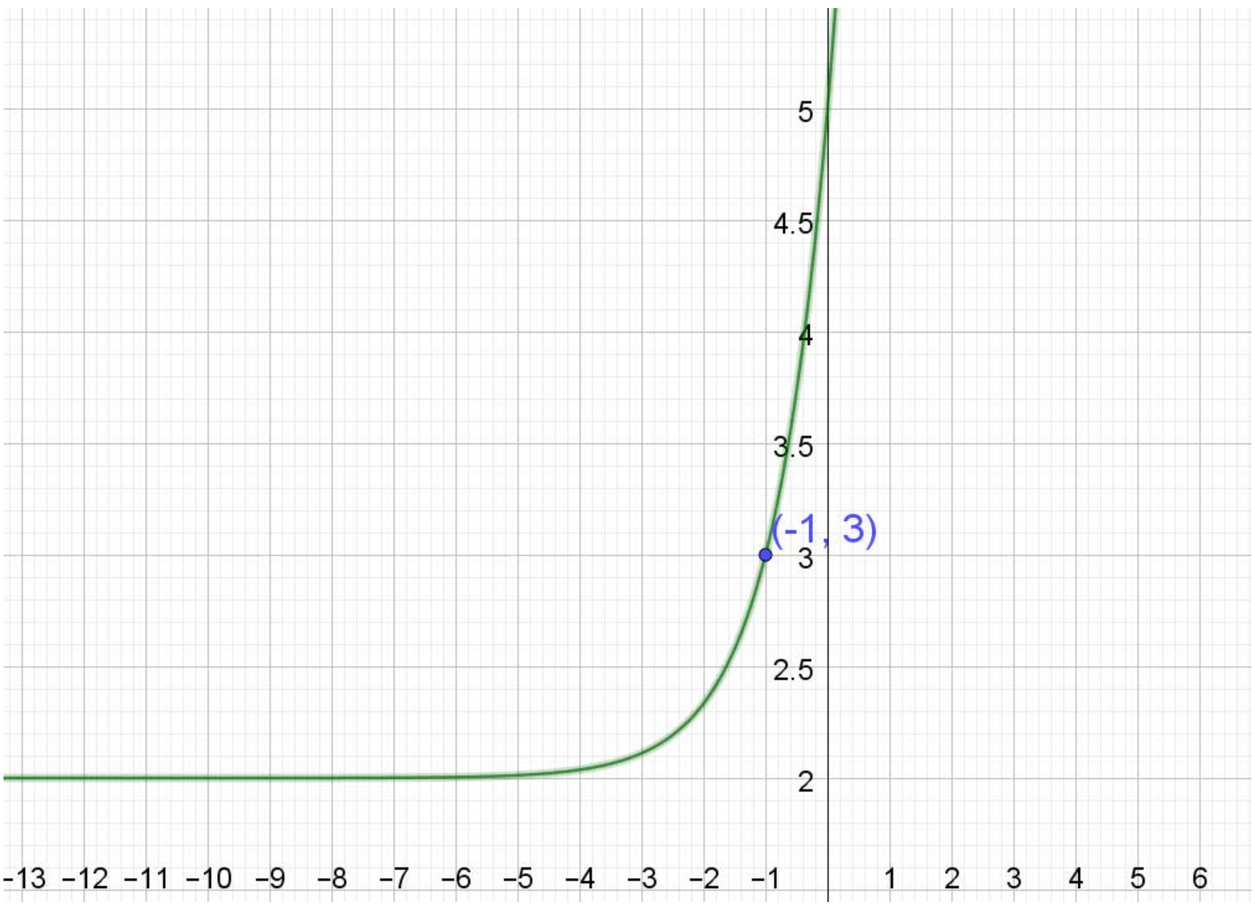
Example: Find the domain, range, and horizontal asymptote of the following functions.

a. $f(x) = 3^{x+1} - 4$

b. $f(x) = 5\left(\frac{1}{2}\right)^x + 1$

c. $f(x) = -3(2^{x+1}) - 5$

Example: Write an equation of an exponential function with the following graph.



Example: Given $f(x) = 2^{x-1}$ and $g(x) = 5e^{10x}$. Evaluate $(g \circ f)(1)$ and $(g \circ f)(0)$.

Logarithmic Functions

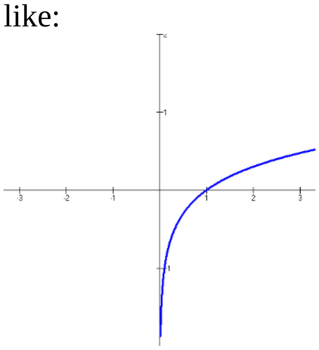
Exponential functions are one-to-one; therefore, they are invertible. The inverse function of the exponential function with base *b* is called the **logarithmic function with base *b***.

The function $f(x) = \log_b x$ is the **logarithmic function with base *b*** with $x > 0, b > 0$ and $b \neq 1$.
Note: The argument (inside) of a logarithmic function must be positive!

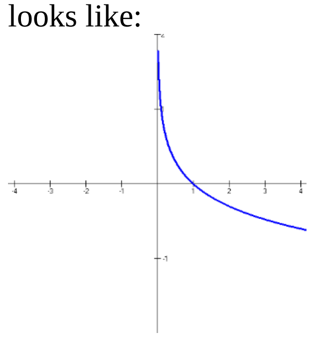
Domain: $(0, \infty)$
Range: $(-\infty, \infty)$
Key Point: (1,0)

The Graph of a Logarithmic Function

If $b > 1$, the graph of $f(x) = \log_b x$ looks like:



If $0 < b < 1$, the graph of $f(x) = \log_b x$ looks like:



Both graphs have a vertical asymptote of x = 0 (the y-axis).

Example: Evaluate $f^{-1}(0)$ if $f(x) = 8^{-1-x} - 32$

Example: Evaluate $f^{-1}(-1)$ if $f(x) = 5^{1-x} - 1$

Logarithm Rules

$y = \log_b x$ is equivalent to $b^y = x$.

Example: $\log_b 1 = 0$

For $b > 0, b \neq 1, x > 0, y > 0$:

Inverse Property of Logarithms

1. $\log_b b^x = x$
2. $b^{\log_b x} = x$

The Product Rule

$\log_b(xy) = \log_b x + \log_b y$

The Quotient Rule

$\log_b \left(\frac{x}{y}\right) = \log_b x - \log_b y$

The Power Rule

$\log_b x^y = y \log_b x$

Example: Evaluate, if possible.

a. $\log_7 49$

b. $\log 1000$

c. $\log_2(-8)$

d. $\log 0$

e. $\ln e^{-3}$

f. $\log_7 \sqrt{7}$

g. $\log_\pi \pi^{-\sqrt{e}}$

h. $\log_3\left(\frac{1}{9}\right)$

Example: Find the domain and vertical asymptotes of the following functions.

a. $\ln(3-2x)$

b. $\log_3(x^2-x-6)$

Example: Write the following logarithm as a sum of logarithms with no products, powers or quotients.

$$\ln\left[\frac{x^4(x-5)^3}{\sqrt{x+3}}\right]$$

Example: Rewrite the following expression as a single logarithm.

$$3\log_7(x-2)-\frac{1}{5}\log_7(x^2-3)-4\log_7(x+5)+1$$

Example: Find $(f \circ g)(x)$ when $f(x) = e^{3x}$ and $g(x) = 5 \ln x$.

Example: Given $f(x) = 5e^{4x-1}$ and $g(x) = \log_2(4x + 1)$, find $f^{-1}(5) + g^{-1}(2)$.

Example: Solve for x.

$$32^{\frac{x}{2}} = 20$$

Example: Solve for x.

$$2^{x-1} = \frac{1}{16}$$

Example: Solve for x.

$$\log(4x - 1) + 3 = 5$$

Example: Solve for x.

$$6e^{x} = 13$$

Example: Find any x- intercepts of the following function.

$$f(x) = \log_6(x+5) + \log_6 x - \log_5 25$$