2312 - Section 4.4 - Trigonometric Expressions and Identities

In this section we are going to practice the algebra involved in working with the trigonometric functions.

Notational Conventions

1. An expression such as $\sin\theta$ really means $\sin(\theta)$.
   An exception to this, however, occurs in expressions such as $\sin(A+B)$, where the parentheses are necessary.
   Example: $\sin\left(\frac{\pi}{4}+\frac{\pi}{2}\right) \neq \sin\frac{\pi}{4}+\frac{\pi}{2}$

2. Parentheses are often omitted in multiplication.
   For example: $(\sin\theta)(\cos\theta)$ is usually written $\sin\theta\cos\theta$.

3. The quantity $(\sin\theta)^n = \sin^n\theta = \underbrace{\sin\theta\sin\theta\ldots\sin\theta}_{n\ times}$.
   Example: $(\sin\theta)^2 = \sin^2\theta = \sin\theta\sin\theta$.
   $\sin^2\theta \neq \sin\theta^2$

In simplifying expressions, it may be useful to use the following identities.

Basic Trigonometric Identities

Reciprocal Identities

1. $\sec\theta = \frac{1}{\cos\theta}, \cos\theta \neq 0$
   $\csc\theta = \frac{1}{\sin\theta}, \sin\theta \neq 0$
   $\cot\theta = \frac{1}{\tan\theta}, \tan\theta \neq 0$

2. $\frac{\sin\theta}{\cos\theta} = \tan\theta$; $\frac{\cos\theta}{\sin\theta} = \cot\theta$

Pythagorean Identities

3. $\sin^2\theta + \cos^2\theta = 1$
   $\tan^2\theta + 1 = \sec^2\theta$
   $\cot^2\theta + 1 = \csc^2\theta$

Example: Simplify.

$$(\sin\alpha - \cos\alpha)^2 + 2\sin\alpha\cos\alpha$$

Recall: $(a+b)^2 = a^2 + 2ab + b^2$
$(a-b)^2 = a^2 - 2ab + b^2$

Example: Simplify.

$$(1 - \cos\alpha)(\csc\alpha + \cot\alpha)$$

Example: Simplify.

$$\sin^4\theta - 2\sin^2\theta + 1$$

Example: Simplify.

$$\sin\alpha\,(\cot\alpha+\tan\alpha)$$

Example: Simplify.

$$\cos^2\alpha+\sin^2\alpha+\cot^2\alpha$$

Example: Simplify.

$$3\cos^2\alpha + \sec^2\alpha + 3\sin^2\alpha - \tan^2\alpha$$

Example: Simplify.

$$\frac{\tan\alpha}{\sec\alpha+1}+\frac{\tan\alpha}{\sec\alpha-1}$$

$\frac{\tan\alpha}{\sec\alpha+1}$ $+\frac{\tan\alpha}{\sec\alpha-1}$

Example: Simplify.

$$1 - \frac{\cos^2 \alpha}{1 - \sin \alpha}$$