$\sin\left(\frac{3\pi}{2}\right) = ?$ $\sin(?) = \frac{\sqrt{3}}{2}$

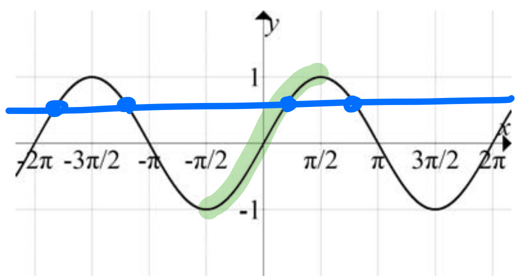
Section 5.4a
Inverse Trigonometric Functions

We now want to evaluate inverse trig functions. With these problems, instead of giving you the angle and asking you for the value, you'll be given the value and ask be asked **what angle gives you that value**; however, we have some restrictions.

Recall the that for an inverse (one-to-one) function, its graph must pass the horizontal line test.

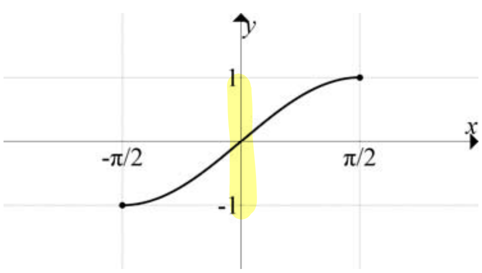
Let's begin with sine.

The function sin(x) is graphed below. Notice that this graph does not pass the horizontal line test; therefore, it does not have an inverse.



Restricted Sine Function and It's Inverse

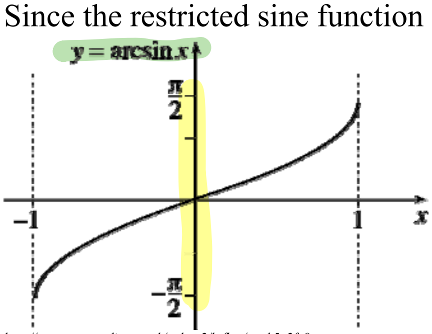
However, if we restrict it from $x = -\frac{\pi}{2}$ to $x = \frac{\pi}{2}$ then we have created the **"Restricted" sine function** and it's one-to-one.



Domain: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Range: [-1, 1]

Since the restricted sine function is one-to-one, it has an inverse $f(x) = \sin^{-1}(x) = \arcsin(x)$.



http://www.met.reading.ac.uk/pplato2/h-flap/math5_2f_8.png

Domain: [-1, 1]

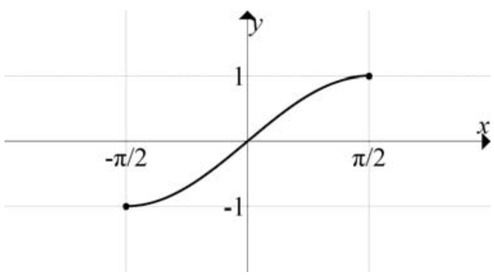
Range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

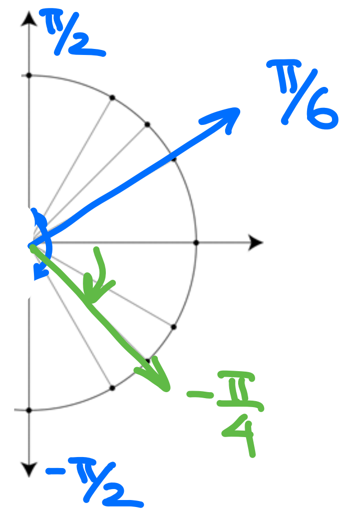
Let's work through a couple of problems.

Remember, we are given the "answer" and are asked to find the angle that gives us that answer, BUT we have some restrictions.

The restrictions when working with arcsine are:

$\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ which are angles from **QUADRANTS 1 AND 4**.





Example 1: Compute $\sin^{-1}\left(\frac{1}{2}\right)$. $= \alpha$

1. $\sin \alpha = \frac{1}{2}$

2. $-\frac{\pi}{2} \le \alpha \le \frac{\pi}{2}$

$\alpha = \frac{\pi}{6}$

$\boxed{\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}}$

Example 2: Compute $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$. $= \alpha$

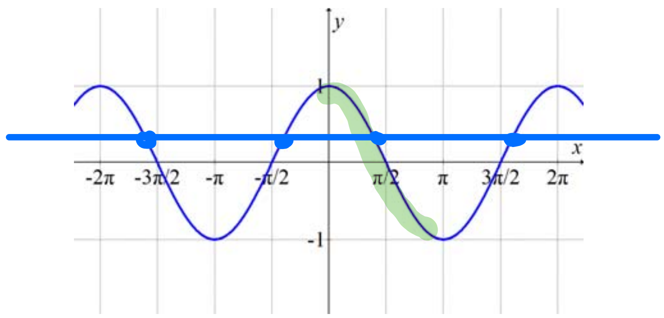
1. $\sin \alpha = -\frac{\sqrt{2}}{2}$

2. α is in QI or Q IV

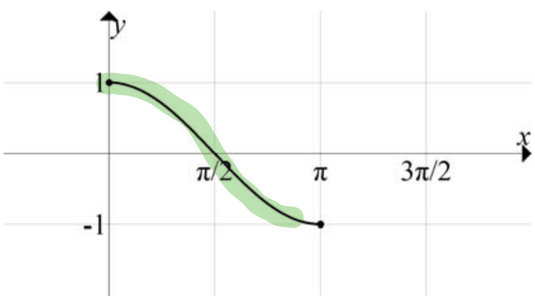
$\boxed{\alpha = -\frac{\pi}{4}}$

Restricted Cosine Function and It's Inverse

The function cos(x) is graphed below. Notice that this graph does not pass the horizontal line test; therefore, it does not have an inverse.



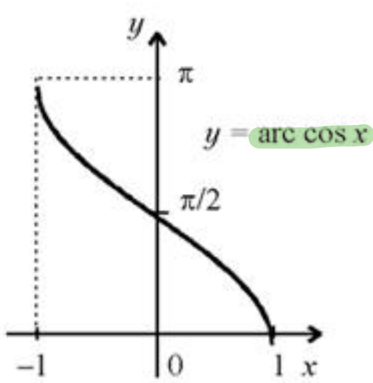
However, if we restrict it from $x = 0$ to $x = \pi$ then we have created the **"Restricted" cosine function** and it's one-to-one.



Domain: $[0, \pi]$

Range: [-1, 1]

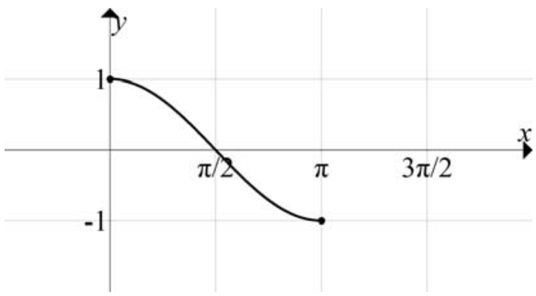
Since the restricted cosine function is one-to-one, it has an inverse $f(x) = \cos^{-1}(x) = \arccos(x)$.

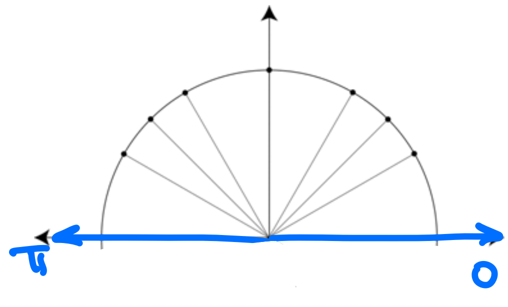


Domain: [-1, 1]

Range: $[0, \pi]$

The restrictions when working with arccosine are:
$[0, \pi]$ which are angles from **QUADRANTS 1 AND 2**.





Example 3: Compute $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$. $= \alpha$

1. $\cos \alpha = -\frac{\sqrt{2}}{2}$

2. $0 \le \alpha \le \pi$

$\boxed{\alpha = \frac{3\pi}{4}}$

Example 4: Compute $\cos^{-1}(0)$. $= \alpha$

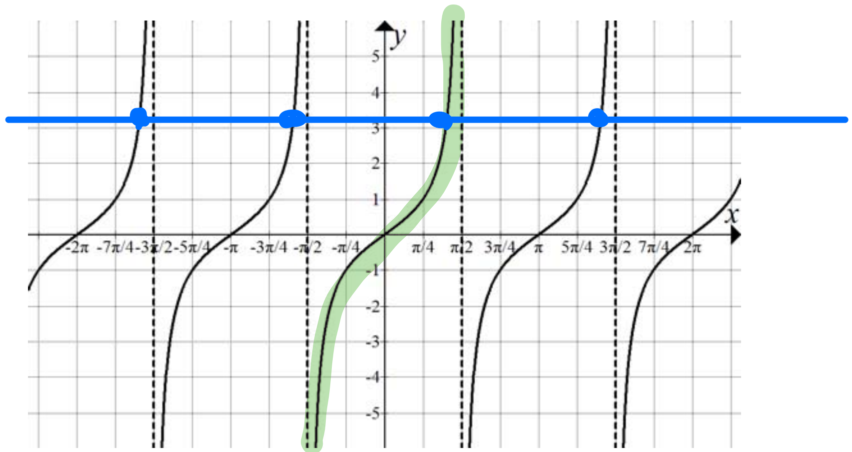
1. $\cos \alpha = 0$

2. $0 \le \alpha \le \pi$

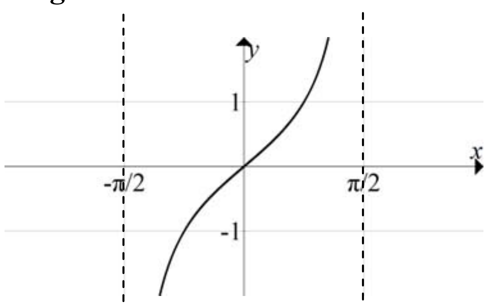
$\boxed{\alpha = \frac{\pi}{2}}$

Restricted Tangent Function and It's Inverse

The function tan(x) is graphed below. Notice that this graph does not pass the horizontal line test; therefore, it does not have an inverse.



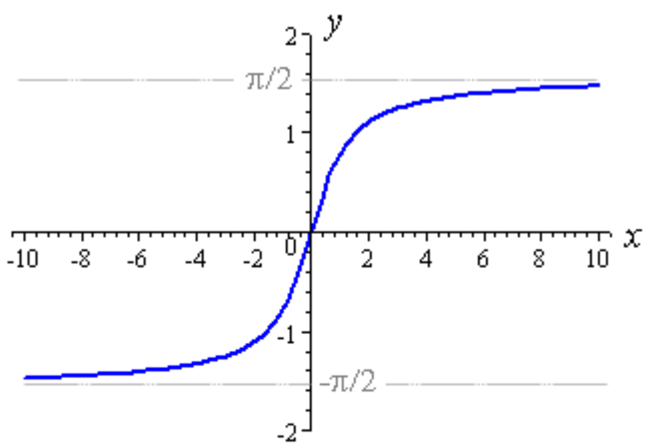
However, if we restrict it from $x = -\frac{\pi}{2}$ to $x = \frac{\pi}{2}$ then we have created the **"Restricted" tangent function** and it's one-to-one.



Domain: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Range: $(-\infty, \infty)$

Since the restricted tangent function is one-to-one, it has an inverse $f(x) = \tan^{-1}(x) = \arctan(x)$.

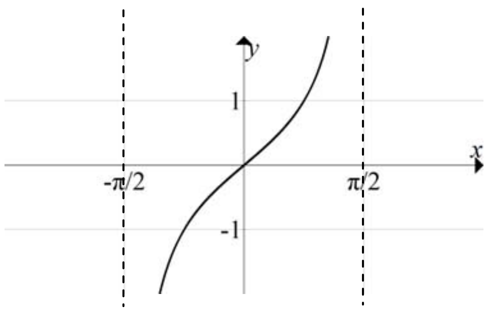


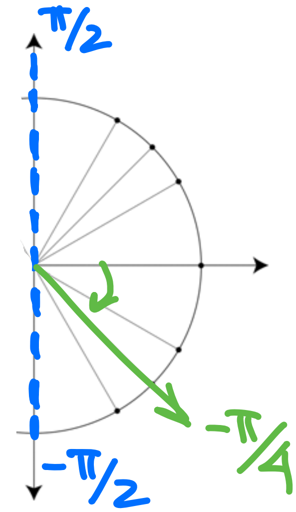
Domain: $(-\infty, \infty)$

Range: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

The restrictions when working with arctangent are:

$\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ which are angles from **QUADRANTS 1 AND 4**.





Example 5: Compute $\arctan(-1)$. $= \alpha$

1. $\tan \alpha = -1$
2. $-\frac{\pi}{2} < \alpha < \frac{\pi}{2}$

$\boxed{\alpha = -\frac{\pi}{4}}$

Example 6: Compute $\tan^{-1}(\sqrt{3})$. $= \alpha$

1. $\tan \alpha = \sqrt{3}$
2. $-\frac{\pi}{2} < \alpha < \frac{\pi}{2}$

$\boxed{\alpha = \frac{\pi}{3}}$

The other inverse trigonometric functions, $y = \cot^{-1}(x)$, $y = \csc^{-1}(x)$, and $y = \sec^{-1}(x)$, can be defined in a manner similar to the inverse trigonometric functions shown above, that is, by restricting the domains of the cotangent, cosecant, and secant functions, respectively, so that the resulting functions are one-to-one.