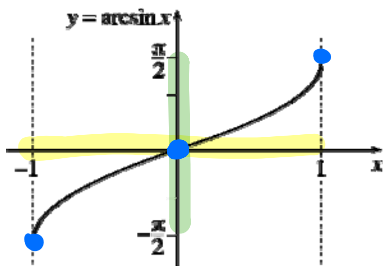
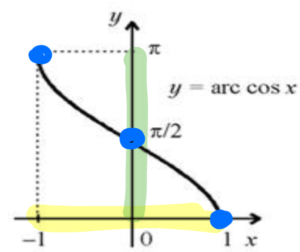
Section 5.4b
Inverse Trigonometric Functions

Recall the following graphs from Section 5.4a.

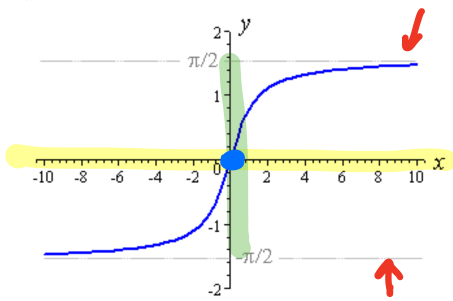
$f(x) = \sin^{-1}(x) = \arcsin(x)$



$f(x) = \cos^{-1}(x) = \arccos(x)$

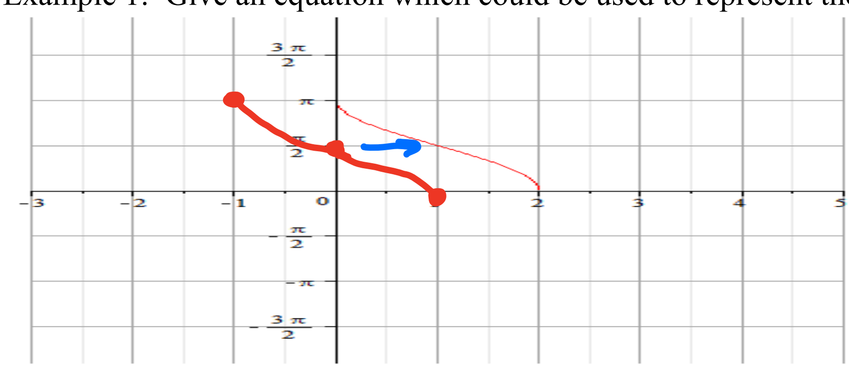


$f(x) = \tan^{-1}(x) = \arctan(x)$.



We can apply simply transformation rules to these types of graphs too.

Example 1: Give an equation which could be used to represent the given graph.



a. $f(x) = \tan^{-1}(x-1)$ b. $f(x) = \sin^{-1}(x+1)$ c. $f(x) = \cos^{-1}(x-1)$
d. $f(x) = \sin^{-1}(x-1)$ e. $f(x) = \cos^{-1}(x+1)$

Example 2: Which of the following points is on the graph of $y = \tan\left(\frac{x}{2}\right) - 2$?

a. $\left(-\frac{\pi}{2}, -1\right)$ Substitute: $-1 \stackrel{?}{=} \tan\left(\frac{-\frac{\pi}{2}}{2}\right) - 2$

$-1 \stackrel{?}{=} \tan\left(-\frac{\pi}{4}\right) - 2$

$-1 \stackrel{?}{=} -1 - 2 \qquad -1 \neq -3$

b. $\left(-\frac{\pi}{2}, -2\right)$ Substitute:

c. $\left(-\frac{\pi}{2}, -3\right)$ Substitute:

Example 3: Which of the following points is on the graph of $y = \arcsin(x+2) - \pi$?

a. $(0, 0)$ Substitute: $0 \stackrel{?}{=} \arcsin(0+2) - \pi$

$0 \neq \arcsin(2) - \pi$

2 is NOT in the domain of $\sin^{-1}$

$D = [-1, 1]$

b. $\left(-3, -\frac{3\pi}{2}\right)$ Substitute:

$-\frac{3\pi}{2} \stackrel{?}{=} \arcsin(-3+2) - \pi$

$-\frac{3\pi}{2} \stackrel{?}{=} \arcsin(-1) - \pi$

$-\frac{3\pi}{2} = -\frac{\pi}{2} - \pi$ ✓

$\arcsin(-1) = \alpha$

$\sin\alpha = -1$ α is in QI or IV

$\alpha = -\frac{\pi}{2}$

c. $(-4, 0)$ Substitute:

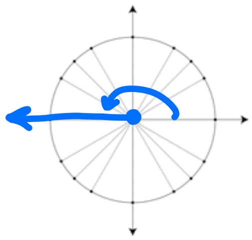
$0 \stackrel{?}{=} \arcsin(-4+2) - \pi$

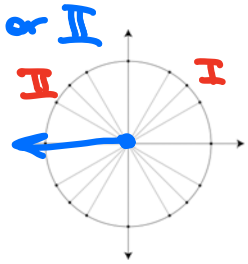
$0 \neq \arcsin(-2) - \pi$ NOT defined

Example 4: Evaluate each of the following.

a. $\arccos(\cos(\pi))$ $= \arccos(-1) = \pi$

1. $\cos \alpha = -1$
2. α is from QI or II

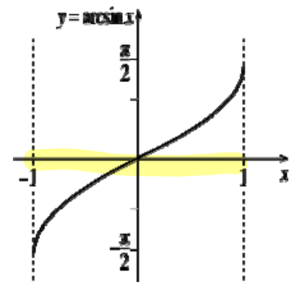




b. $\sin(\arcsin(-2))$.

undefined

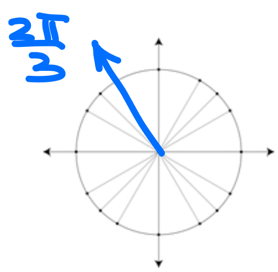
Recall:

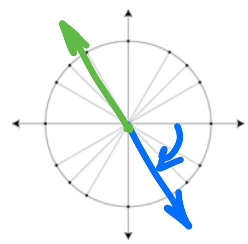


c. $\arctan\left(\tan\left(\frac{2\pi}{3}\right)\right)$ $= \arctan(-\sqrt{3}) = -\frac{\pi}{3}$

1. $\tan\left(\frac{2\pi}{3}\right) = -\sqrt{3}$

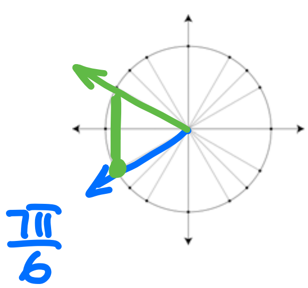
2. $\tan \alpha = -\sqrt{3}$
α is from QI or IV

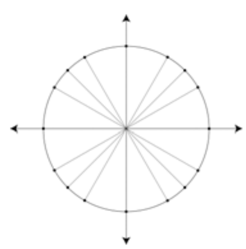




d. Evaluate $\arccos\left(\cos\left(\frac{7\pi}{6}\right)\right)$. $= \arccos\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}$

1. $\cos \alpha = -\frac{\sqrt{3}}{2}$
2. α is in QI or II





α

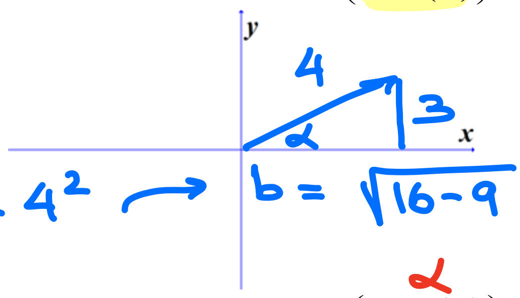
Example 5: Evaluate $\cos\left(\sin^{-1}\left(\frac{3}{4}\right)\right) = \cos\alpha$

$= \boxed{\frac{\sqrt{7}}{4}}$

1. $\sin^{-1}\left(\frac{3}{4}\right) = \alpha$

$\sin\alpha = \frac{3}{4}$

α is in QI or IV



$b^2 + 3^2 = 4^2 \rightarrow b = \sqrt{16-9} = \sqrt{7}$

α

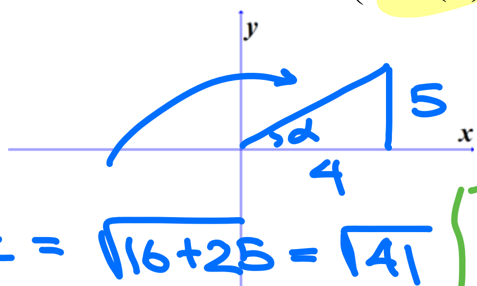
Example 6: Evaluate $\cos\left(\tan^{-1}\left(\frac{5}{4}\right)\right)$. $= \cos\alpha$

$= \frac{4}{\sqrt{41}} = \boxed{\frac{4\sqrt{41}}{41}}$

1. $\tan^{-1}\left(\frac{5}{4}\right) = \alpha$

$\tan\alpha = \frac{5}{4}$

α is in QI or IV



$c = \sqrt{16+25} = \sqrt{41}$

$\sin\left(\tan^{-1}\left(-\frac{5}{4}\right)\right) = \sin\alpha$

$\tan\alpha = -\frac{5}{4}$ $= -\frac{5}{\sqrt{41}} = -\frac{5\sqrt{41}}{41}$

α is Q IV

Models

As we know, trigonometric functions repeat their behavior. Breathing normally, brain waves during deep sleep are just a couple of examples that can be described using a sine function.

Example 7: The current I, in amperes, flowing through an ac (alternating current) circuit at time t is modeled by $I = 210\sin\left(40\pi t - \frac{\pi}{3}\right)$ where $t \geq 0$. Find its:

a. amplitude. A =

b. period. $\frac{2\pi}{B} =$

c. horizontal shift.
$\frac{C}{B} =$

d. maximum value.