Section 6.1
Sum and Difference Formulas

$\sin(A+B) = \sin A\cos B + \cos A\sin B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$\sin(A-B) = \sin A\cos B - \cos A\sin B$

$\cos(A-B) = \cos A\cos B + \sin A\sin B$

$$\tan(A+B) = \frac{\tan A + \tan B}{1-\tan A\tan B}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1+\tan A\tan B}$$

Example 1: Use the appropriate angle-sum formula to simplify the following expression.

a. $\sin(15^o)\cos(75^o)+\cos(15^o)\sin(75^o)$

$= \sin(15^\circ + 75^\circ)$

$= \sin(90^\circ) = 1$

b. $\cos\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{5}\right)-\sin\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{5}\right)$

$= \cos\left(\frac{\pi}{3}+\frac{\pi}{5}\right)$

$= \cos\left(\frac{8\pi}{15}\right)$

Example 2: If $\sin x = \alpha$ then $\sin(\pi - x) = \alpha$. $\boxed{\text{True}}$ or False?

$\underbrace{\sin(\pi - x)}_{A-B} = \sin(\pi)\cos(x) - \cos(\pi)\sin(x)$

$= 0\cdot\cancel{\cos(x)} - (-1)\sin(x)$

$= \sin(x) = a$

Example 3: $\tan\left(\frac{\pi}{2}-x\right)=\cot x$, True or False?

Be careful when verifying whether a statement involving tan or cot is true or false. This is because tan and cot come from sin/cos or cos/sin, respectively.

Start with the left-hand side…there's more to work with there.

$$\tan\left(\frac{\pi}{2}-x\right) = \frac{\sin\left(\frac{\pi}{2}-x\right)}{\cos\left(\frac{\pi}{2}-x\right)}$$

$$= \frac{\sin\left(\frac{\pi}{2}\right)\cos x - \cos\left(\frac{\pi}{2}\right)\sin x}{\cos\left(\frac{\pi}{2}\right)\cos x + \sin\left(\frac{\pi}{2}\right)\sin x}$$

$$= \frac{(1)\cos(x) - \cancel{(0)\sin(x)}}{\cancel{(0)\cos(x)} + (1)\sin(x)} = \frac{\cos x}{\sin x} = \cot(x)$$

Q III Q IV

Example 4: Let $\tan\alpha = \frac{5}{2}$ with $\pi < \alpha < \frac{3\pi}{2}$ and $\cos\beta = \frac{1}{2}$, with $\frac{3\pi}{2} < \beta < 2\pi$. Find $\cos(\alpha+\beta)$ and $\tan(\alpha-\beta)$.

Recall: $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$$= \left(\frac{-2}{\sqrt{29}}\right)\left(\frac{1}{2}\right) - \left(\frac{-5}{\sqrt{29}}\right)\left(-\frac{\sqrt{3}}{2}\right)$$

$$= \frac{-2}{2\sqrt{29}} - \frac{5\sqrt{3}}{2\sqrt{29}} = \boxed{\frac{-2-5\sqrt{3}}{2\sqrt{29}}} \cdot \frac{\sqrt{29}}{\sqrt{29}} = \frac{-2\sqrt{29}-5\sqrt{87}}{58}$$

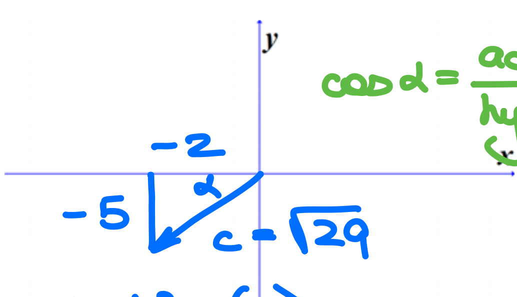
$\tan(\alpha-\beta)$.

Recall: $\tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta} = \frac{\frac{5}{2} - \left(-\frac{\sqrt{3}}{1}\right)}{1 + \frac{5}{2}\left(-\frac{\sqrt{3}}{1}\right)}$

$$= \frac{\frac{5}{2} + \frac{\sqrt{3}\cdot 2}{1\cdot 2}}{\frac{2\cdot 1}{2\cdot 1} - \frac{5\sqrt{3}}{2}} = \frac{\left(\frac{5+2\sqrt{3}}{\cancel{2}}\right)\cancel{2}}{\left(\frac{2-5\sqrt{3}}{\cancel{2}}\right)\cancel{2}} = \boxed{\frac{5+2\sqrt{3}}{2-5\sqrt{3}}}$$

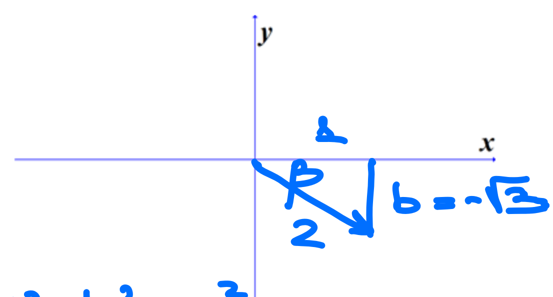
For alpha: $\tan\alpha = \frac{5}{2} = \frac{opp}{adj}$

$\cos\alpha = \frac{adj}{hyp} = \frac{-2}{\sqrt{29}}$



$c^2 = (-5)^2 + (-2)^2 = 29$

For beta: $\cos\beta = \frac{1}{2}$



$1^2 + b^2 = 2^2$

$b^2 = 4 - 1$

$\sqrt{b^2} = \sqrt{3}$ $b = -\sqrt{3}$

$c = \sqrt{29}$

$\sin\left(\frac{5\pi}{12}\right)$

$\frac{5\pi}{12} = A + B$ $\qquad$ $\frac{5\pi}{12} = A - B$

$\frac{\pi}{6} = \frac{2\pi}{12}$ $\qquad$ $\frac{\pi}{4} = \frac{3\pi}{12}$

$\frac{\pi}{3} = \frac{4\pi}{12}$ $\qquad$ $\frac{\pi}{2} = \frac{6\pi}{12}$

$\frac{5\pi}{12} = \frac{\pi}{6} + \frac{\pi}{4}$

$$\sin\left(\frac{5\pi}{12}\right) = \sin\left(\overset{A}{\frac{\pi}{6}} + \overset{B}{\frac{\pi}{4}}\right)$$

$$= \sin\left(\frac{\pi}{6}\right)\cos\left(\frac{\pi}{4}\right) + \cos\left(\frac{\pi}{6}\right)\sin\left(\frac{\pi}{4}\right)$$

$$= \frac{1}{2}\cdot\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}$$

$\sqrt{a}\cdot\sqrt{b} = \sqrt{a\cdot b}$

$$= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \boxed{\frac{\sqrt{2}+\sqrt{6}}{4}}$$