Section 6.2
Double-Angle and Half-Angle Formulas

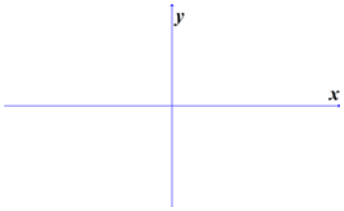
Double-Angle Formulas

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$
$$\cos(2\theta) = 2\cos^2\theta - 1$$
$$\cos(2\theta) = 1 - 2\sin^2\theta$$

$$\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$$

Example 1: Suppose $\csc\theta = -\frac{5}{4}$ and $\pi < \theta < \frac{3\pi}{2}$.



a. Find $\sin(2\theta)$.

Recall: $\sin(2\theta) = 2\sin\theta\cos\theta$

b. Find $\cos(2\theta)$.

Recall: $\cos(2\theta) = \cos^2\theta - \sin^2\theta$

Half-Angle Formulas

$$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{2}} \qquad \cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos\theta}{2}} \qquad \tan\left(\frac{\theta}{2}\right) = \frac{\sin\theta}{1+\cos\theta}$$

Note: In the half-angle formulas the $\pm$ symbol is intended to mean either positive or negative but not both, and the sign before the radical is determined by the quadrant in which the angle $\frac{\theta}{2}$ terminates.

c. Find $\cos\left(\frac{\theta}{2}\right)$. *Recall:* $\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos\theta}{2}}$ and $\pi < \theta < \frac{3\pi}{2}$

d. Find $\sin\left(\frac{\theta}{2}\right)$. *Recall:* $\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{2}}$ and $\pi < \theta < \frac{3\pi}{2}$

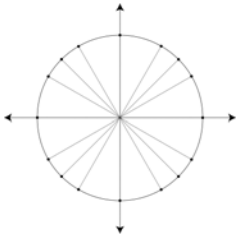
Example 2: Use the half-angle formula to calculate $\sin\left(\frac{5\pi}{12}\right)$. *Recall:* $\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{2}}$

a. Which quadrant does $\frac{5\pi}{12}$ live in?

b. Is $\sin\left(\frac{5\pi}{12}\right)$ positive or negative?

c. Rewrite $\sin\left(\frac{5\pi}{12}\right)$ so that it's in the form $\sin\left(\frac{\theta}{2}\right)$ and then calculate.

What does θ equal?



Example 3: Use the half-angle formula to calculate $\cos\left(\frac{13\pi}{8}\right)$. *Recall:* $\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos\theta}{2}}$

a. Which quadrant does $\left(\frac{13\pi}{8}\right)$ live in?

b. Is $\cos\left(\frac{13\pi}{8}\right)$ positive or negative?

c. Rewrite $\cos\left(\frac{13\pi}{8}\right)$ so that it's in the form $\cos\left(\frac{\theta}{2}\right)$ and then calculate.

What does θ equal?