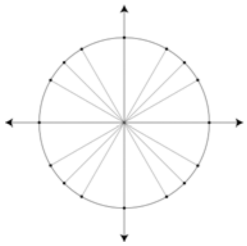
Section 6.3
Solving Trigonometric Equations

An equation that contains a trigonometric expression is called a **trigonometric equation**. As we know trigonometric functions repeat their behavior. What if we wanted answers to questions like: "When will the moon look exactly like it did last night at 9:30pm?" and "When will my breathing be exactly as it is right now?". The questions to these models can be answered by solving trigonometric equations.

Example 1: Let $\tan(x) = -1$.

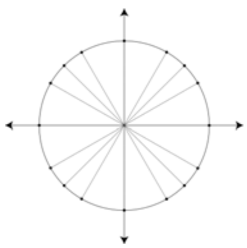
a. Find all solutions in the interval $[0, 2\pi)$

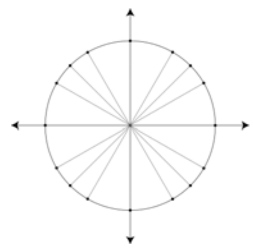
b. Find all solutions.



Example 2: Let $2\sin^2 x - 3\sin x = -1$.

a. Solve the equation in the interval $[0, 2\pi)$.

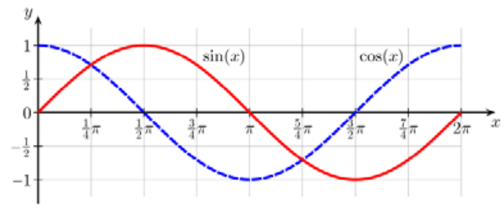




b. Find all solutions.

Example 3: Solve the equation $\sin(2x) = -15\cos x$ in the interval $\left[0, \dfrac{\pi}{2}\right]$.

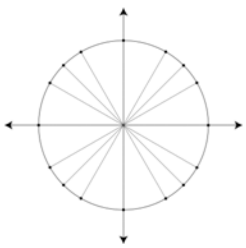
Recall: $\sin(2x) = 2\sin x\cos x$



In solving trig equations if you ever get

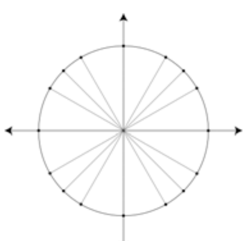
- $\sin x > 1$
- $\sin x < -1$
- $\cos x > 1$
- $\cos x < -1$

then you have no solution.



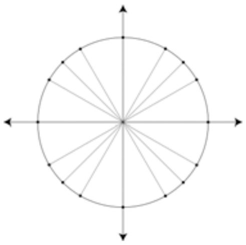
Try this one: Solve the equation $10\sin^2 x = 10\cos^2 x$ in the interval $[0, 2\pi)$.

Recall: $\sin^2 x + \cos^2 x = 1$



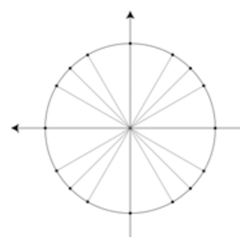
Example 4: Find all solutions of $\sec^2 x = 1$ in the interval $\left[-\frac{\pi}{2}, \frac{5\pi}{2}\right]$.

Recall: $\sin^2 x + \cos^2 x = 1$ divided by $\cos^2 x$ produces $\tan^2 x + 1 = \sec^2 x$

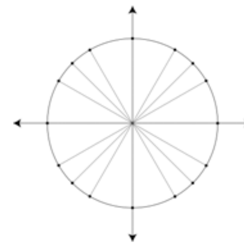


Equations Involving Multiples of an Angle

Example 5: Find all solutions of $\tan(3x) = -1$ in the interval $\left[0, \frac{2\pi}{3}\right)$.

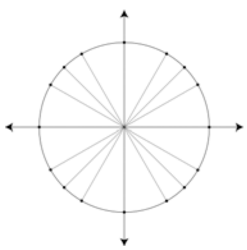


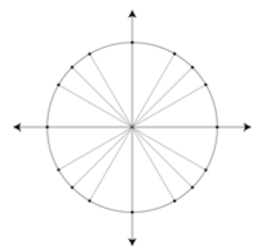
Example 6: Find all solutions in $\left[-\frac{1}{3}, \frac{2}{3}\right]$ for $\sin\left(\pi x + \frac{\pi}{3}\right) = 1$



Try these:

I. Find all solutions of $2\sin\left(\dfrac{x}{2}\right) = -1$ in the interval $[-\pi, \pi]$.





II. Find all solutions in $\left[0, \frac{\pi}{2}\right]$ for $2\cos\left(2x - \frac{\pi}{6}\right) = \sqrt{3}$