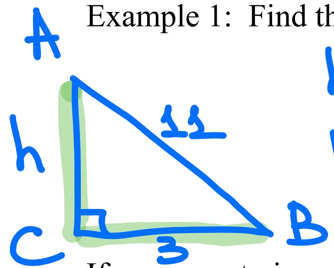
Section 7.2
Area of a Triangle

Area of a triangle

Given the base, b, and the height, h, of a triangle we can calculate its area by applying the formula: $\boxed{A=\frac{1}{2}bh}$

Example 1: Find the area of triangle ABC, where $m\angle C=90^o$, a = 3mm and c = 11mm.



$h^2+3^2=11^2$

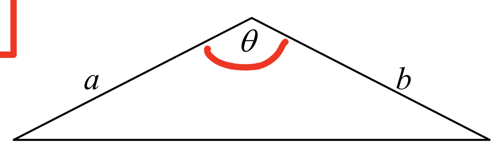
$h=\sqrt{121-9}$

$=\sqrt{112}=\sqrt{4\cdot4\cdot7}=4\sqrt{7}$

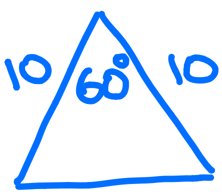
$A=\frac{1}{\cancel{2}}(3)(\cancel{4}^{2}\sqrt{7})$

$=\boxed{6\sqrt{7}\ mm^2}$

If we are not given the base and the height of the triangle, but given two sides and the angle ***between*** them then we can still calculate its area by applying the formula: $\boxed{A=\frac{1}{2}ab\sin\theta}$



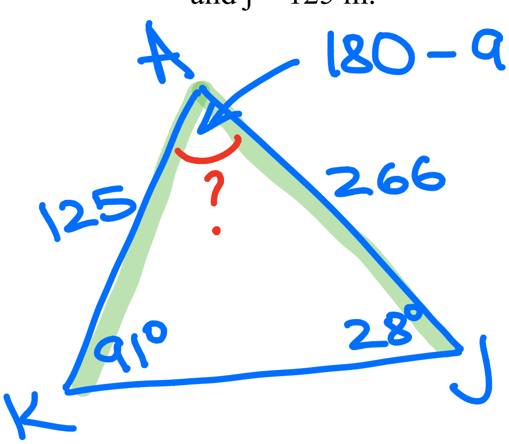
Example 2: Find the area of the equilateral triangle with side lengths 10 ft.



$A=\frac{1}{2}(10)(10)\sin(60^\circ)$

$=\frac{1}{\cancel{2}}(\cancel{10}^{5})(\cancel{10}^{5})\cdot\frac{\sqrt{3}}{\cancel{2}}=25\sqrt{3}\ ft^2$

Example 3: Find the area of triangle JAK with angle K = 91°, angle J = 28°, k = 266 m, and j = 125 m.



$180-91-28=61^\circ$

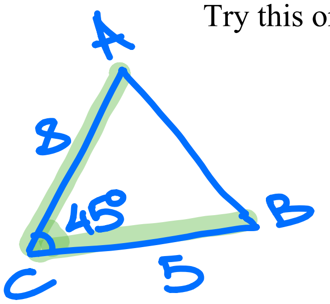
$A=\frac{1}{\cancel{2}}(125)(\cancel{266}^{133})\sin(61^\circ)$

$=16625\sin(61^\circ)$ test

$=14540.55\ m^2$ quiz

$$A = \frac{1}{2} ab \sin C$$

Try this one: Find the area of triangle ABC with angle C = 45°, a = 5 cm and b = 8 cm.

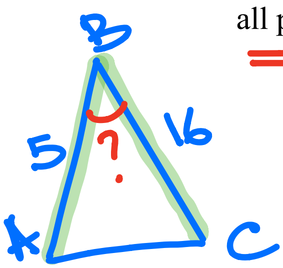


$$A = \cancel{\tfrac{1}{2}}(\cancel{8}^{4})(5)\sin 45°$$

$$= {}^{10}\cancel{20} \cdot \frac{\sqrt{2}}{\cancel{2}} = \boxed{10\sqrt{2}\ \text{cm}^2}$$

Area

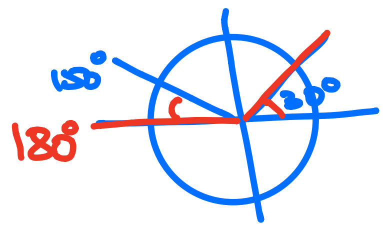
Example 4: If the area of ΔABC is 20 square centimeters, a = 16 cm and c = 5 cm, find all possible measures for angle B.



$$20 = \cancel{\tfrac{1}{2}}(\cancel{16}^{8})(5)\sin B$$

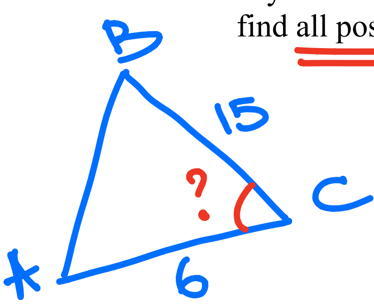
$$20 = 40 \sin B$$

$$\sin B = \frac{20}{40} = \frac{1}{2}$$



$$\boxed{m\angle B = 30° \qquad m\angle B = 150°}$$

Try this one: If the area of triangle ABC is 14 square meters, with a = 15 m and b = 6 m, find all possible measures for angle C. Round answers to the nearest hundredth.



$$A = \frac{1}{2} ab \sin C$$

$$14 = \cancel{\tfrac{1}{2}}(15)(\cancel{6}^{3})\sin C$$

$$14 = 45 \sin C \qquad \sin C = \frac{14}{45}$$

$$m\angle C = \arcsin\left(\frac{14}{45}\right) = 18.13°$$

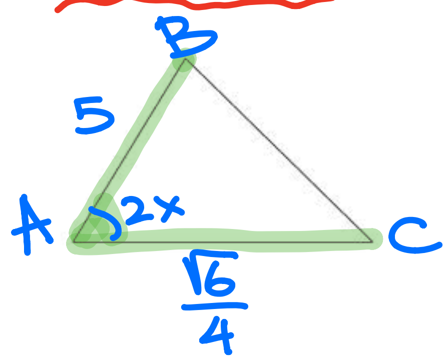
$$m\angle C = 180 - \arcsin\left(\frac{14}{45}\right)$$

$$= 180 - 18.13° = 161.87°$$

Try this one: In Δ ABC, the measure of $\angle A = (2x)^o$, c = 5 in, $b = \frac{\sqrt{6}}{4}$ in and $\sin x = \frac{1}{5}$.

Find the area of Δ ABC. *Hint: The double angle formula for sine will be useful.*

$\sin(2x) = 2\sin x \cos x$

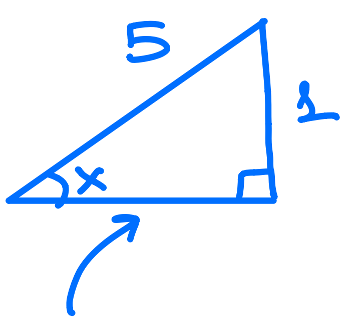


$$A = \frac{1}{2}(5)\left(\frac{\sqrt{6}}{4}\right)\sin(2x)$$

$$= \frac{5\sqrt{6}}{8}\sin(2x)$$

$$= \frac{5\sqrt{6}}{\cancel{8}\,4}\cdot \cancel{2}\sin(x)\cos x$$

$$= \frac{5\sqrt{6}}{4}\sin(x)\cos(x)$$



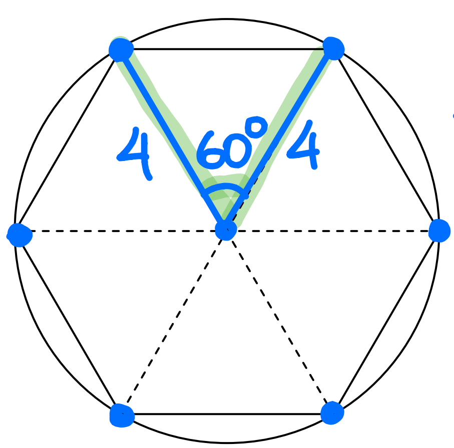
$$\sqrt{25-1} = \sqrt{24}$$

$$= \sqrt{4\cdot 6} = 2\sqrt{6}$$

$$= \frac{\cancel{5}\sqrt{6}}{\cancel{4}\,2}\cdot\frac{1}{\cancel{5}}\cdot\frac{\cancel{2}\sqrt{6}}{5}$$

$$= \frac{6}{10} = \boxed{\frac{3}{5}\text{ in}^2}$$

Example 5: A regular hexagon is inscribed in a circle of radius 4m. Find the area of the hexagon.



Area of hexagon

$$= 6\cdot \text{Area of } \Delta$$

$$= 6\cdot\frac{1}{\cancel{2}}(\cancel{4}\,2)(4)\sin(60^\circ)$$

$$= \frac{\cancel{48}\,24\cdot\sqrt{3}}{\cancel{2}}$$

$$= \boxed{24\sqrt{3}\text{ m}^2}$$