$f(x) = ax^2+bx+c$

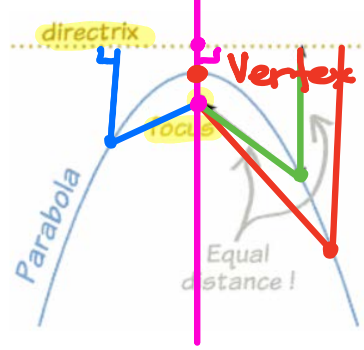
$a>0$ $a<0$

Section 8.1
Parabolas

NOT functions

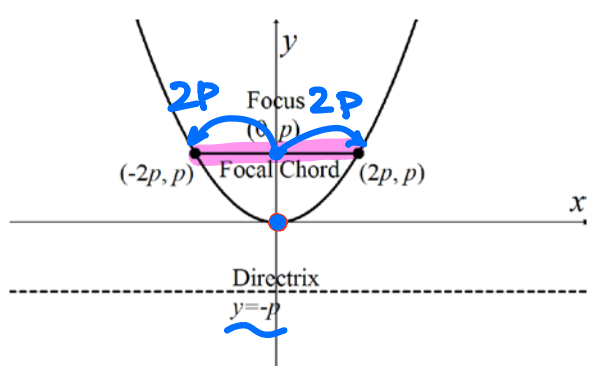
We previously studied parabolas as the graphs of quadratic functions. Now we will look at them as conic sections. There are a few differences. For example, when we studied quadratic functions, we saw that the graphs of the functions could open up or down. As we look at conic sections, we'll see that the graphs of these second degree equations can also open left or right. So, not every parabola we'll look at in this section will be a function.

A **parabola** is the set of all points equally distant from a fixed line and a fixed point not on the line. The fixed line is called the **directrix**. The fixed point is called the **focus**. The axis, or **axis of symmetry**, runs through the focus and is perpendicular to the directrix. The **vertex** is the point halfway between the focus and the directrix.



http://www.mathsisfun.com/geometry/parabola.html

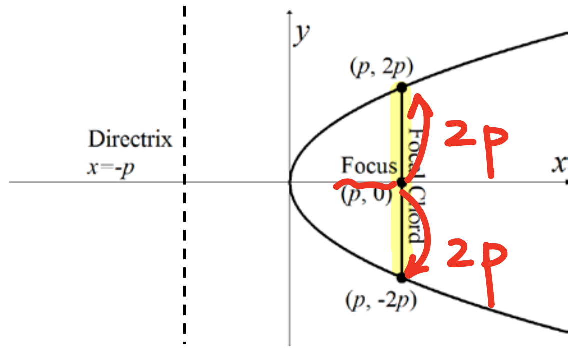
A basic vertical parabola's equation (vertex is at the origin) is of the form: $\boxed{x^2 = 4py}$. Its basic graph is shown below. The focal width is $|4p|$.



$x^2 = 4y$
$4 = 4p$
$p = 1$

$x^2 = 10y$
$10 = 4p$
$p = \frac{10}{4} = \frac{5}{2}$

A basic horizontal parabola's equation (vertex is at the origin) is of the form: $\boxed{y^2 = 4px.}$ Its basic graph is shown below. The focal width is $|4p|$.



Graphing parabolas with vertex at the origin:

- If it has x^2, it's a "vertical" parabola. If it has y^2, it's a "horizontal" parabola.

- Rearrange the equation into the form $y^2 = 4px$ or $x^2 = 4py$. In other words, isolate the squared variable. If 4p is positive, then the parabola opens up. If 4p is negative, then the parabola opens down. left right

- Determine p by setting the absolute value of the coefficient of the variable raised to the first power equal to 4p, and solve.

- Starting at the origin, place the focus p units to the inside of the parabola. Place the directrix p units to the outside of the parabola.

- Use the focal width $|4p|$ ($2p$ on each side) to make the parabola the correct width at the focus.

Graphing parabolas with vertex not at the origin:

- Rearrange the equation into the form $(y-k)^2 = 4p(x-h)$ or $(x-h)^2 = 4p(y-k)$. You may need to complete the square.

- Vertex is (h,k). Follow the steps above, except start at this vertex.

Example 1: Sketch the graph of $x^2 = -4y$ and state each of the following features listed below.

$-4 = 4p \quad p = -1$

Orientation: Down

P-value: -1

Vertex: $(0,0)$

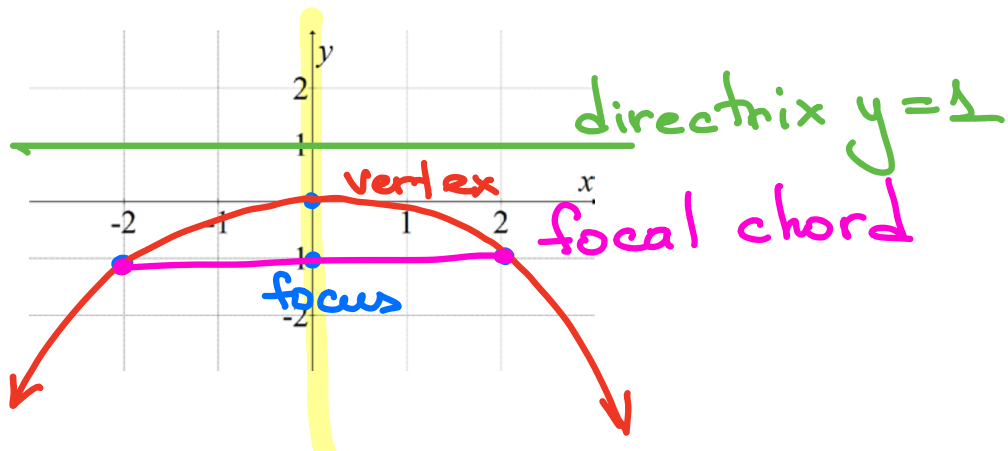
Focus: $(0,p)$ $(0,-1)$

Directrix: $y = -p$ $y = 1$

Focal width: $|4p| = 4$

Axis of symmetry: $x = 0$

Endpoints of the focal chord: $(2,-1)$ & $(-2,-1)$



Example 2: Write $y^2 + 2y + 8x + 17 = 0$ in standard form. Sketch its graph and state each of the following features listed below.

$y^2 + 2y + 1 = -8x - 17 + 1 \qquad \left(\frac{2}{2}\right)^2 = (1)^2 = 1$

$(y+1)^2 = -8x - 16$

$(y+1)^2 = -8(x+2)$

$(y-k)^2 = 4p(x-h)$

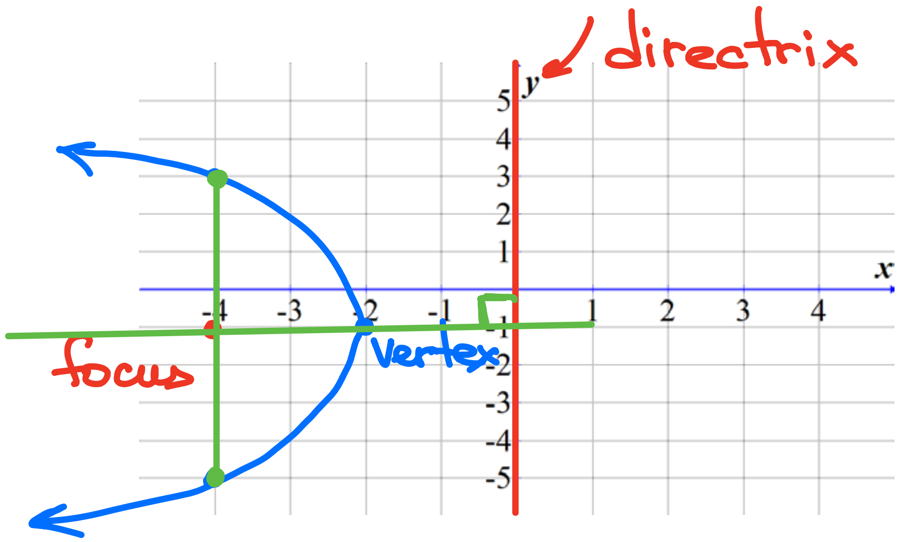
Vertex: (h,k) Vertex: $(-2,-1)$

$4p = -8 \quad p = -2$

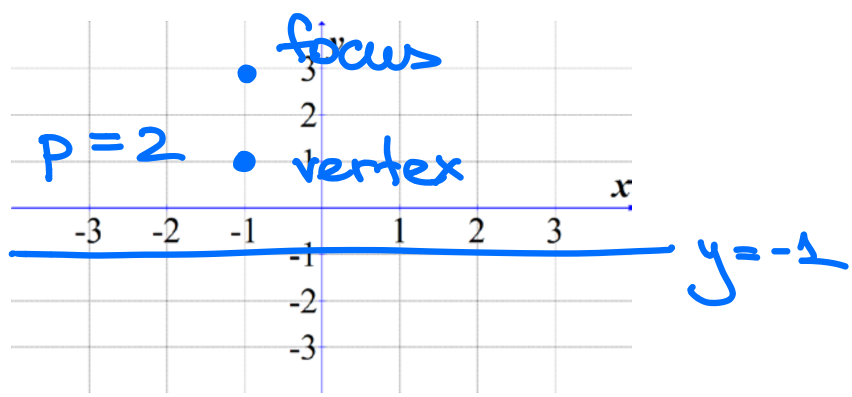
Orientation: left P-value: -2 Vertex: $(-2,-1)$ Focus:

Directrix: $x=0$ Focal width: $|4p|=8$ Axis of symmetry: $y=-1$

Endpoints of the focal chord: $(-4,3)$ & $(-4,-5)$



Example 3: Suppose you know that the focus of a parabola is (-1, 3) and the directrix is the line $y=-1$.



a. Using the given information, which of the following equations will model the parabola?

A. $(y-k)^2 = 4p(x-h)$ B. $(x-h)^2 = 4p(y-k)$.

Vertex: $(\overset{h}{-1}, \overset{k}{1})$

$p=2$

b. Write an equation for the parabola in standard form.

$$\boxed{(x+1)^2 = 8(y-1)}$$