Section 8.2b
Ellipses

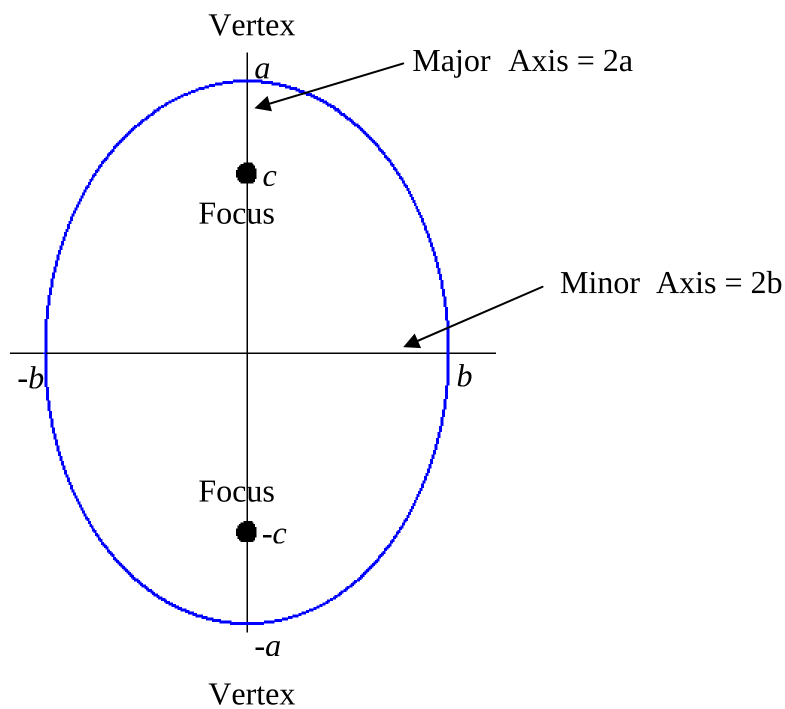
An **ellipse** is the set of all points, the sum of whose distances from two fixed points is constant. Each fixed point is called a **focus** (plural = *foci*).

Basic "Vertical" Ellipse (center is at the origin):

Basic "vertical" ellipse:

Equation: $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1,\ a>b$

Foci: $c^2=a^2-b^2$



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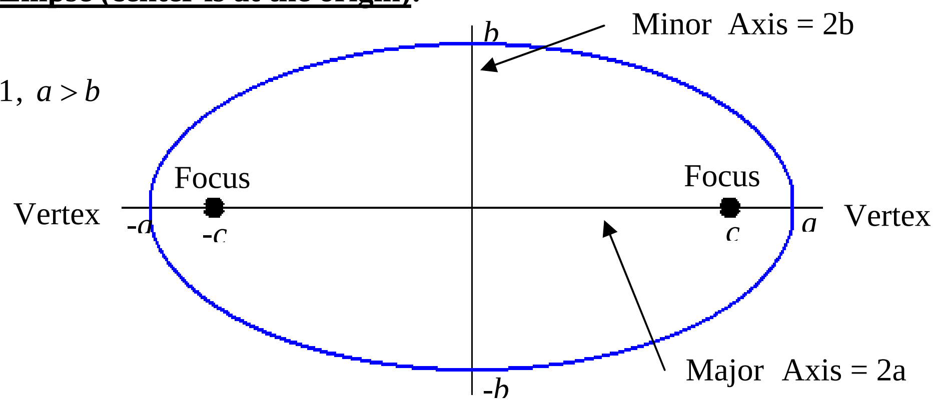


The **eccentricity** provides a numerical measure of how much the ellipse deviates from being a circle. The *eccentricity e* is a number between 0 and 1.

Eccentricity: $e=\frac{c}{a}$

To graph an ellipse with center at the origin:

- Rearrange the equation into the form $\frac{x^2}{number} + \frac{y^2}{number} = 1$.

- If the bigger number is under x^2, it's horizontal. If the bigger number is under y^2, it's vertical.

- Use the square root of the number under x^2 to determine how far to measure in *x*-direction from the center.

- Use the square root of the number under y^2 to determine how far to measure in *y*-direction from the center.

- Draw the ellipse with these measurements. Be sure it is smooth with no sharp corners.

- Determine the location of the foci.
  Formula: $c^2 = a^2 - b^2$ where a^2 and b^2 are the denominators. (Subtract the small denominator from the large denominator to get c^2.) The foci are located *c* units from the center of the long axis.

- The vertices and foci must lie on the Major Axis.

For ellipses:
In the formulas on page 1, "a" is associated with the LARGER DENOMINATOR.

Example 1: Given the ellipse in standard form $\frac{x^2}{9}+\frac{y^2}{16}=1$, sketch its graph and state the features listed below.

Orientation:

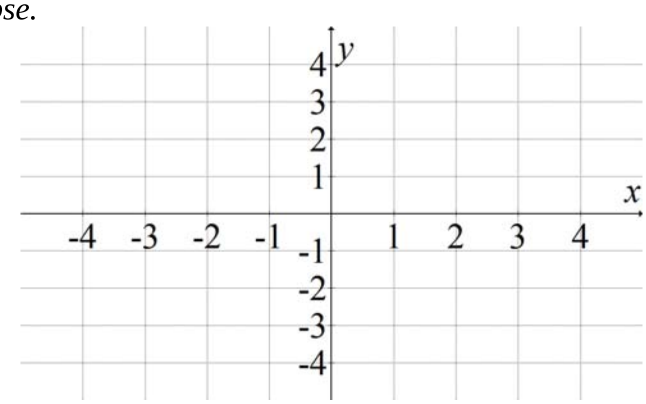
Center:

Recall: For ellipses, "a" is associated with the larger denominator.

x-direction:

y-direction:

Next, draw the ellipse.



Vertices:

Length of Major Axis:

Coordinates of the Major Axis:

Length of Minor Axis:

Coordinates of the Minor Axis:

Foci Formula: $c^2 = a^2 - b^2$

Eccentricity: $e = \frac{c}{a}$

To graph an ellipse with center not at the origin, rearrange the equation into the form $\frac{(x-h)^2}{number}+\frac{(y-k)^2}{number}=1$. You may need to complete the square. Start at the center (*h*, *k*) and graph it as before.

Example 2: Write $4x^2-8x+9y^2-54y=-49$ in standard form. Sketch its graph and state the features listed below.

Orientation:

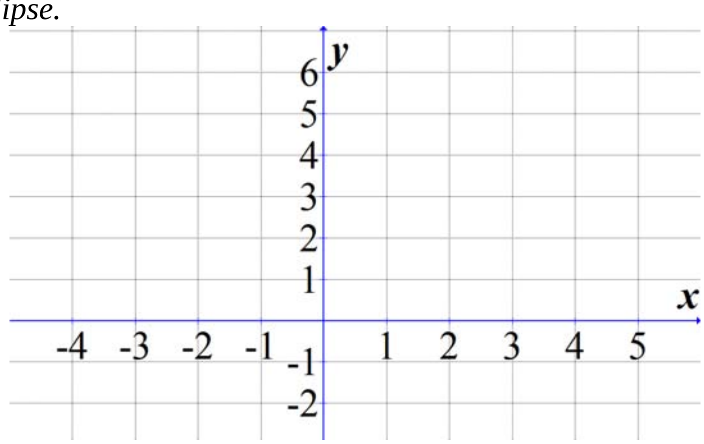
Center:

Recall: For ellipses, "a" is associated with the larger denominator.

x-direction:

y-direction:

Next, draw the ellipse.



Vertices:

Length of Major Axis:

Coordinates of the Major Axis:

Length of Minor Axis:

Coordinates of the Minor Axis:

Foci Formula: $c^2 = a^2 - b^2$

Eccentricity: $e = \frac{c}{a}$