Section 8.2b
Ellipses

An **ellipse** is the set of all points, the sum of whose distances from two fixed points is constant. Each fixed point is called a **focus** (plural = *foci*).

Basic "Vertical" Ellipse (center is at the origin):

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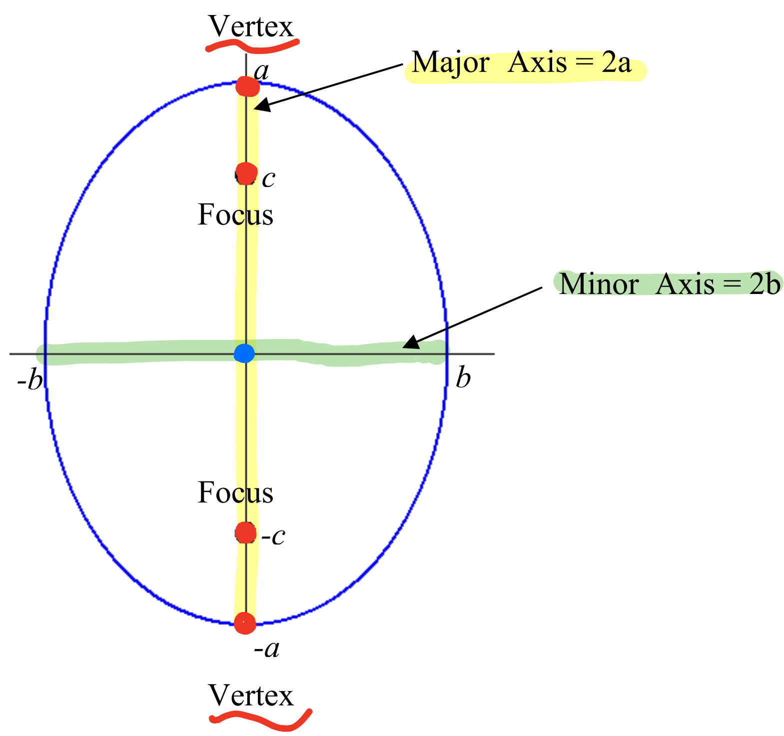
Equation: $\boxed{\frac{x^2}{b^2}+\frac{y^2}{a^2}=1}$, $a>b$

Foci: $c^2=a^2-b^2$



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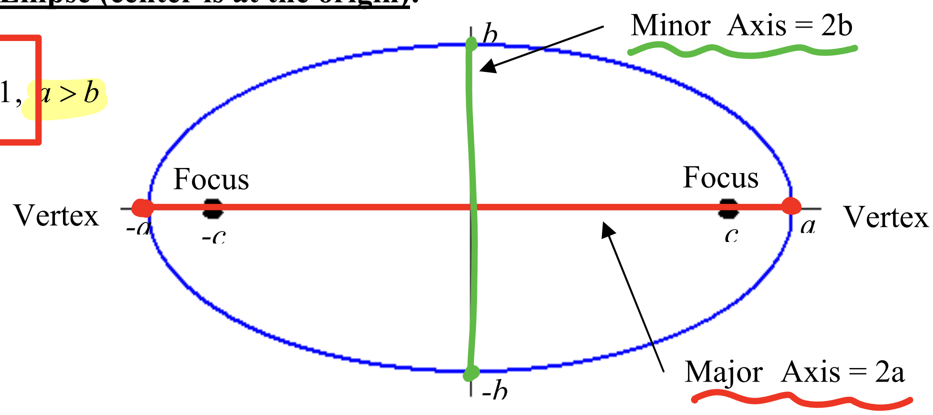


The **eccentricity** provides a numerical measure of how much the ellipse deviates from being a circle. The *eccentricity e* is a number between 0 and 1.

Eccentricity: $\boxed{e=\frac{c}{a}}$

To graph an ellipse with center at the origin:

- Rearrange the equation into the form $\dfrac{x^2}{number}+\dfrac{y^2}{number}=1$.

- If the bigger number is under x^2, it's horizontal. If the bigger number is under y^2, it's vertical.

- Use the square root of the number under x^2 to determine how far to measure in x-direction from the center.

- Use the square root of the number under y^2 to determine how far to measure in y-direction from the center.

- Draw the ellipse with these measurements. Be sure it is smooth with no sharp corners.

- Determine the location of the foci.
  Formula: $c^2 = a^2 - b^2$ where a^2 and b^2 are the denominators. (Subtract the small denominator from the large denominator to get c^2.) The foci are located c units from the center of the long axis.

- The vertices and foci must lie on the Major Axis.

For ellipses:
In the formulas on page 1, "a" is associated with the LARGER DENOMINATOR.

Example 1: Given the ellipse in standard form $\frac{x^2}{9}+\frac{y^2}{16}=1$, sketch its graph and state the features listed below.

Orientation: Vertical

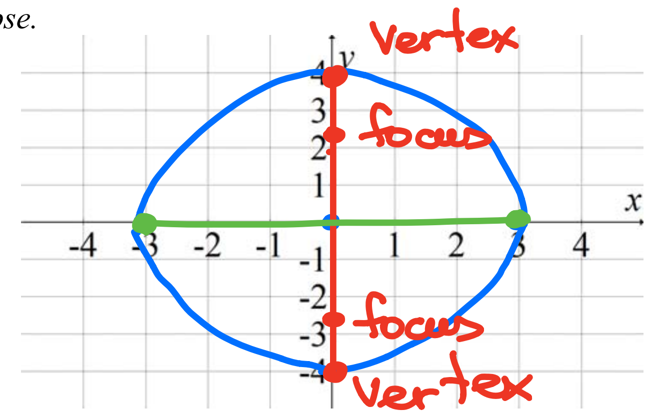
Center: $(0,0)$

Recall: For ellipses, "a" is associated with the larger denominator.

x-direction: $\sqrt{9} = 3 = b$

y-direction: $\sqrt{16} = 4 = a$

Next, draw the ellipse.



Vertices: $(0,4)$ & $(0,-4)$

Length of Major Axis: $=2(4)=8$
$2a$

Coordinates of the Major Axis: $(0,4)$ & $(0,-4)$

Length of Minor Axis: $=2(3)=6$
$2b$

Coordinates of the Minor Axis: $(-3,0)$ & $(3,0)$

Foci Formula: $c^2 = a^2 - b^2 = 16-9=7$

$\sqrt{c^2}=\sqrt{7}$ $\quad c=\sqrt{7}$ $\quad (0,\sqrt{7})$ & $(0,-\sqrt{7})$

$\sqrt{4}<\sqrt{7}<\sqrt{9}$

$2<\sqrt{7}<3$

Eccentricity: $e=\frac{c}{a}$

$e=\frac{\sqrt{7}}{4}$

To graph an ellipse with center not at the origin, rearrange the equation into the form $\frac{(x-h)^2}{number}+\frac{(y-k)^2}{number}=1$. You may need to complete the square. Start at the center (h, k) and graph it as before.

Example 2: Write $4x^2-8x+9y^2-54y=-49$ in standard form. Sketch its graph and state the features listed below.

$$4(x^2-2x+1)+9(y^2-6y+9)=-49+4+81$$

$$\frac{4(x-1)^2}{36}+\frac{9(y-3)^2}{36}=\frac{36}{36}$$

For x: $\left(\frac{-2}{2}\right)^2=1$

For y: $\left(\frac{-6}{2}\right)^2=9$

$$\frac{(x-1)^2}{9}+\frac{(y-3)^2}{4}=1$$

Orientation: Horizontal

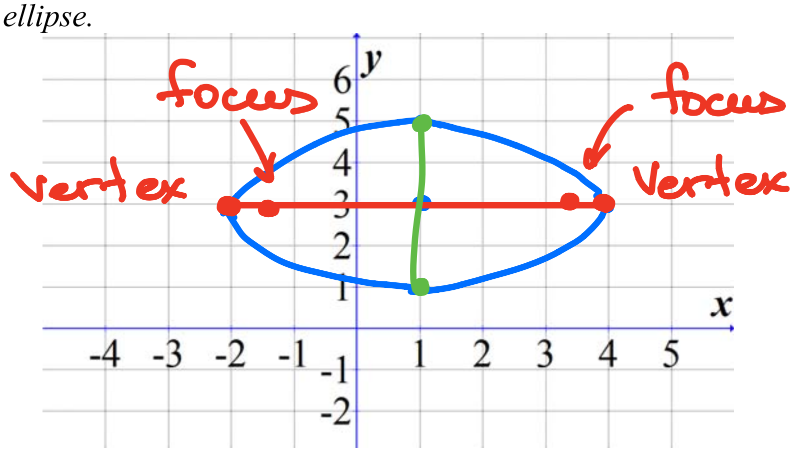
Center: $(1, 3)$

Recall: For ellipses, "a" is associated with the larger denominator.

x-direction: $\sqrt{9}=3=a$

y-direction: $\sqrt{4}=2=b$

Next, draw the ellipse.



Vertices: (4,3) & (-2,3)

Length of Major Axis: $2(3)=6$
$2a$

Coordinates of the Major Axis:
(4,3) & (-2,3)

Length of Minor Axis: $2(2)=4$
$2b$

Coordinates of the Minor Axis:
(1,1) & (1,5)

Foci Formula: $c^2 = a^2 - b^2$ $= 9-4=5$

$c^2 = 5$ $\quad c = \sqrt{5}$

$(1+\sqrt{5}, 3)$ & $(1-\sqrt{5}, 3)$

$\sqrt{4} < \sqrt{5} < \sqrt{9}$

$2 < \sqrt{5} < 3$

Eccentricity: $e = \dfrac{c}{a}$ $= \dfrac{\sqrt{5}}{3}$