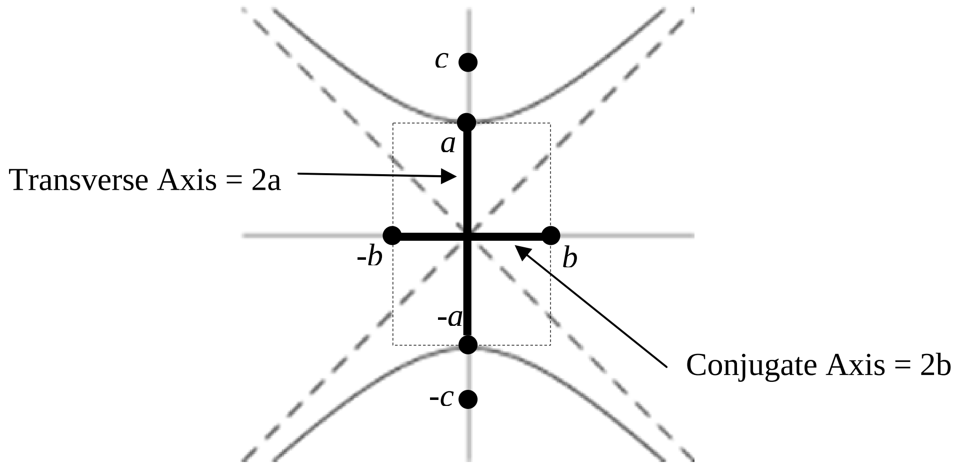
Section 8.3
Hyperbolas

A **hyperbola** is the set of all points, the difference of whose distances from two fixed points is constant. Each fixed point is called a **focus** (plural = *foci*). The **focal axis** is the line passing through the foci.

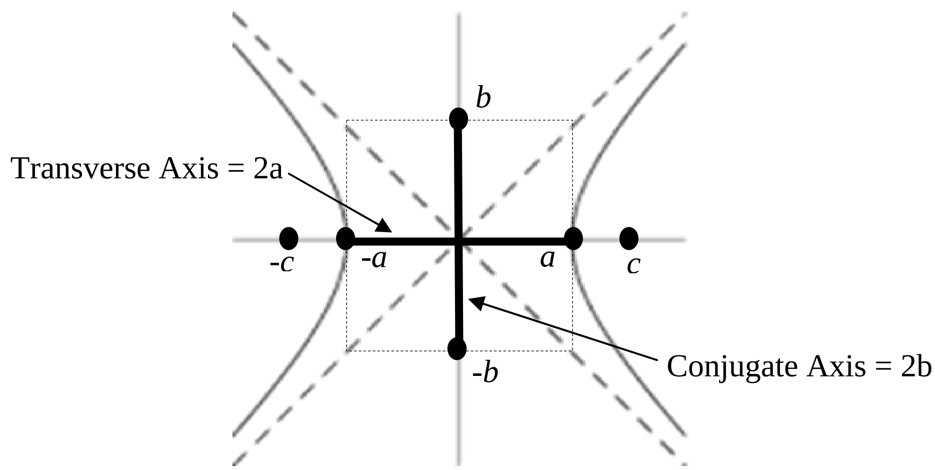
Basic "Vertical" Hyperbola (center is at the origin):

Equation: $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ Foci Formula: $c^2 = a^2 + b^2$



Basic "Horizontal" Hyperbola (center is at the origin:

Equation: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ Foci Formula: $c^2 = a^2 + b^2$



To graph a hyperbola with center at the origin:

- Rearrange the equation into the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ or $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$.
- If the x^2 is positive, it's horizontal. If the y^2 is positive, it's vertical.
- Use the square root of the number under x^2 to determine how far to measure in *x*-direction from the center.
- Use the square root of the number under y^2 to determine how far to measure in *y*-direction from the center.
- Draw a rectangle with these measurements. Then draw two diagonals through the center, from one corner of the rectangle to the opposite corner. These are the asymptotes.
- Use the dimensions of the rectangle to determine the slopes of the asymptotes and write their equations.
- Put the vertices at the edge of the box on the correct axis. Then draw a hyperbola, making sure it approaches the asymptotes smoothly.
- Find the foci. Formula: $c^2 = a^2 + b^2$ where a^2 and b^2 are the denominators. The foci are located *c* units from the center, on the same axis as the vertices.

For hyperbolas:
In the formulas at the top, "a" is associated with the POSITIVE TERM!

Example 1: Given the hyperbola in standard form $\frac{x^2}{36} - \frac{y^2}{4} = 1$, sketch its graph and state each of the following features listed below.

Orientation:

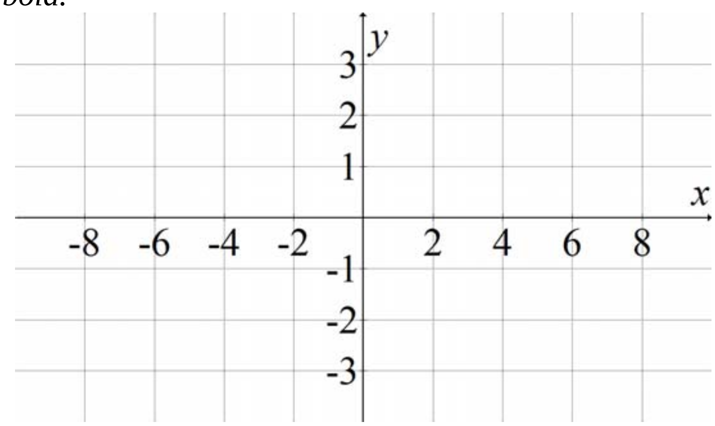
Center:

Recall: For hyperbolas, "a" is associated with the positive term.

x-direction:

y-direction:

Next, draw the hyperbola.



Asymptotes:

Vertices:

Foci Formula: $c^2 = a^2 + b^2$

Foci:

Length of the Transverse Axis:

Coordinates of the Transverse Axis:

Length of the Conjugate Axis:

Coordinates of the Conjugate Axis:

To graph a hyperbola with center not at the origin:

- Rearrange the equation into the form $\dfrac{(x-h)^2}{a^2}-\dfrac{(y-k)^2}{b^2}=1$ or $\dfrac{(y-k)^2}{a^2}-\dfrac{(x-h)^2}{b^2}=1$
- Start at the center (h,k) and then graph it as before.
- To write down the equations of the asymptotes, start with the equations $y=\pm mx$ and replace x with $x - h$ and replace y with $y - k$.

Example 2: Write $y^2-25x^2+8y-9=0$ in standard form. Sketch its graph and state each of the following features listed below.

Orientation:

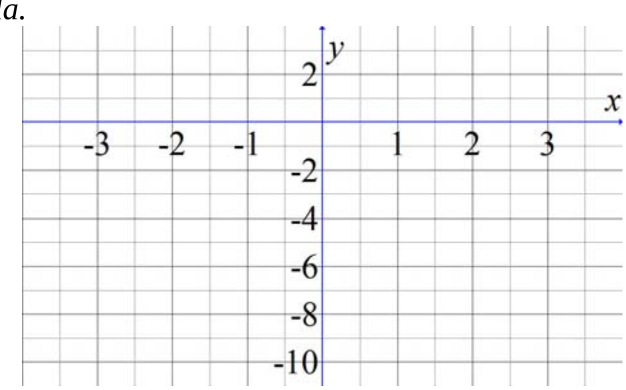
Recall: For hyperbolas, "a" is associated with the positive term.

Center:

x-direction:

y-direction:

Next, draw the hyperbola.



Asymptotes:

Vertices:

Foci Formula: $c^2 = a^2 + b^2$

Foci:

Length of the Transverse Axis:

Coordinates of the Transverse Axis:

Length of the Conjugate Axis:

Coordinates of the Conjugate Axis:

Example 3: Use the given features to answer the following questions.
Center: (6, 8)
Length of transverse axis = 16
Length of conjugate axis = 6
Horizontal transverse axis

a. Which of the following equations will model the hyperbola?

A. $\dfrac{(x-h)^2}{a^2} - \dfrac{(y-k)^2}{b^2} = 1$

B. $\dfrac{(y-k)^2}{a^2} - \dfrac{(x-h)^2}{b^2} = 1$

b. Write an equation for the hyperbola in standard form.