Chapter 8
Systems: Identify Equations, Point of Intersection of Equations

Recall the following equations:

Parabola: $(y-k)^2 = 4p(x-h)$ or $(x-h)^2 = 4p(y-k)$

Circle: $(x-h)^2 + (y-k)^2 = r^2$ $\quad C = (h, k)$

Ellipse: $\dfrac{(x-h)^2}{number} + \dfrac{(y-k)^2}{number} = 1$

Hyperbola: $\dfrac{(x-h)^2}{a^2} - \dfrac{(y-k)^2}{b^2} = 1$ or $\dfrac{(y-k)^2}{a^2} - \dfrac{(x-h)^2}{b^2} = 1$

Example 1: Identify each conic.

a. $12x = y^2$ parabola

b. $\dfrac{(x-2)^2}{9} - \dfrac{(y+2)^2}{16} = 1$ hyperbola

c. $\dfrac{(x+4)^2}{4} + \dfrac{(y-1)^2}{9} = 1$ ellipse

d. $\dfrac{(x-2)^2}{5} + \dfrac{(y+2)^2}{5} = 1$ circle

$(x-2)^2 + (y+2) = 5$ $\quad r^2$ $\quad r = \sqrt{5}$

If the equation is written in general form, $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, with only minimal work, you can determine if an equation in this form is a circle, an ellipse, a parabola or a hyperbola.

If A, B and C are not all 0, and if the graph if not degenerate (point, line or two lines), then:

- The graph is a circle if $B^2 - 4AC < 0$ and $A = C$.
- The graph is an ellipse if $B^2 - 4AC < 0$ and $A \neq C$.
- The graph is a parabola if $B^2 - 4AC = 0$.
- The graph is a hyperbola if $B^2 - 4AC > 0$.

Example 2: Identify the following conic: $2x^2 - 8y^2 - 6x - 16y - 25 = 0$

$A = 2$
$B = 0$
$C = -8$

$B^2 - 4AC = (0)^2 - 4(2)(-8) > 0$

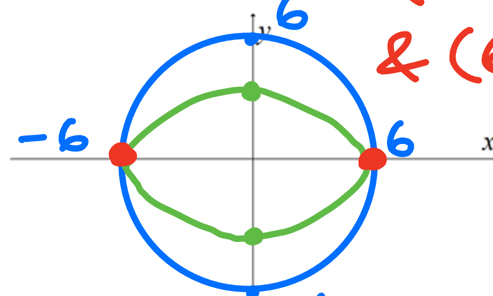
hyperbola

Systems of Second Degree Equations

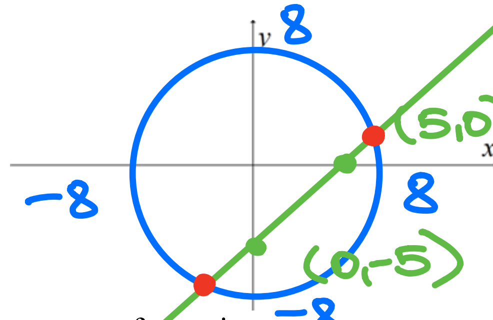
When we graph two conic sections or a conic section and a line on the same coordinate planes, their graphs may contain points of intersection. We want to be able to find the points of intersection. To do this, we may either graph the system of equations or solve a system of equations.

Example 3: Determine the number of points of intersection by graphing.

a. $x^2 + y^2 = 36$ and $\frac{x^2}{36} + \frac{y^2}{9} = 1$



b. Determine the number of points of intersection by graphing.
$x^2 + y^2 = 64$ and $y = x - 5$



Example 4: Solve each of the following the systems of equations.

a. $f(x) = -2x^2 + 8x - 5$
$g(x) = 6x - 5$

b. $5x^2 + 4y^2 = 9$

$(6x^2 - y^2 = 5)\ 4$

$$5x^2 + 4y^2 = 9$$
$$+\ 24x^2 - 4y^2 = 20$$
$$29x^2 = 29$$
$$x^2 = 1$$
$$x = \pm 1$$

$x = 1$

$$5(1)^2 + 4y^2 = 9$$
$$5 + 4y^2 = 9$$
$$4y^2 = 4$$
$$y^2 = 1$$
$$y = \pm 1$$

$x = -1$

$$5(-1)^2 + 4y^2 = 9$$
$$5 + 4y^2 = 9$$
$$y = \pm 1$$

$(1, 1)$ & $(1, -1)$ & $(-1, 1)$ & $(-1, -1)$

Try this one: Find any points of intersection of:
$xy = 3$ and $x - y = -2$ → $x = y - 2$

$$(y-2)y = 3$$
$$y^2 - 2y = 3$$
$$y^2 - 2y - 3 = 0$$
$$(y-3)(y+1) = 0$$
$$y = 3 \quad y = -1$$

Sub. back

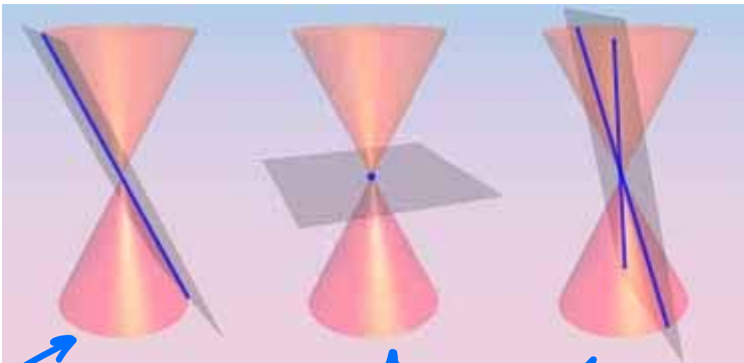
$y = 3 \quad x = 3 - 2 = 1$

$(1, 3)$

$y = -1 \quad x = -1 - 2 = -3$

$(-3, -1)$

Degenerate Conic Sections



An example of each follows.

I. $(x-3)^2+(y+1)^2=0$ represents a point (3, -1). Looks like it could be a circle equation, but $r=0$.

II. $9x^2-4y^2=0$ represents 2 lines. Looks like it could be a hyperbola, but right hand-side is 0, not 1.

Solve for y:

$$4y^2=9x^2$$
$$y^2=\frac{9x^2}{4}$$
$$y=\pm\frac{3x}{2}$$

III. $(y-5)^2=0$ represents one line.

Solve for y:

$$y-5=0$$
$$y=5$$

Another example would be: $(x+2)^2=0$.

$x+2=0 \quad x=-2$

IV. $2x^2+3y^2=-1$ represents nothing, no graph, no point, no line(s). Looks like it could be an ellipse, but right hand-side is -1, not 1.