MATH 2312 Test 3 Review

Students are responsible for reserving a seat for the test using **Schedule Exams** tab on CASA BEFORE the first day of test while seats are available. You take your test in a **CASA Testing Center**.

If you miss your scheduled test, then check the scheduler to see if there are other available times and if you can reschedule it. Your instructor does not control the scheduler, cannot add test slots, and/or reschedule your test – make sure you do not miss your reserved time.

FAQ

- ✓ **What is covered on the test?** Chapter 5, 6.1, 6.2.
- ✓ **How many questions are there?** 17 Multiple-choice questions to be completed in 60 minutes.
- ✓ **What calculator can I use on exams?** No calculators or other software are allowed on exams – study accordingly.
- ✓ **Will there be a formula sheet provided?** Yes! It will be a link. See the formula sheet on the next page. You canNOT have a copy of a unit circle while taking the test – make sure you know your unit circle.
- ✓ **How do I study for the best result?**
  1. Begin with this review. SOLVE a blank copy of this review after it is discussed in class!
  2. Take Practice Test 3 BEFORE your test! It is for practice AND extra credit. 5% of your best score will be added to your Test 3 score. Retake Practice Test 3 several times to strengthen your skills.
  3. Review online quizzes and homework.

While this review covers most of the “skills” you should be practicing it, is not intended to be the only resource you use when you prepare for the test.

KNOW UNIT CIRCLE!
Know inverse trig values.

TEST 3 Formula Sheet

Math 1330
Formula Sheet

sin (s + t) = sin s cos t + cos s sin t

sin (s – t) = sin s cos t – cos s sin t

cos (s + t) = cos s cos t – sin s sin t

cos (s – t) = cos s cos t + sin s sin t

$$\tan(s+t) = \frac{\tan s + \tan t}{1 - \tan s \tan t}$$

$$\tan(s-t) = \frac{\tan s - \tan t}{1 + \tan s \tan t}$$

sin(2t) = 2sin t cos t

$$\cos(2t) = \cos^2 t - \sin^2 t$$

$$\sin\frac{s}{2} = \pm\sqrt{\frac{1-\cos s}{2}}$$

$$\cos\frac{s}{2} = \pm\sqrt{\frac{1+\cos s}{2}}$$

$$\tan\frac{s}{2} = \frac{\sin s}{1+\cos s}$$

$\cos(x+2\pi k)=\cos x$ $\quad$ $\tan(x+\pi k)=\tan x$

$\sin(x+2\pi k)=\sin x$ $\quad$ $\cot(x+\pi k)=\cot x$

1. Evaluate the following:

$\frac{32}{3} = 10+\frac{2}{3}$

a) $\sin\left(\frac{32\pi}{3}\right) = \sin\left(10\pi+\frac{2}{3}\pi\right) = \sin\left(\frac{2}{3}\pi\right) = \frac{\sqrt{3}}{2}$

b) $\cos\left(\frac{41\pi}{6}\right) = \cos\left(6\pi+\frac{5\pi}{6}\right) = \cos\left(\frac{5\pi}{6}\right) = -\frac{\sqrt{3}}{2}$

c) $\tan\left(\frac{43\pi}{4}\right) = \tan\left(10\pi+\frac{3\pi}{4}\right) = \tan\left(\frac{3\pi}{4}\right) = -1$

d) $\cot\left(\frac{21\pi}{4}\right) = \cot\left(5\pi+\frac{\pi}{4}\right) = \cot\left(\frac{\pi}{4}\right) = 1$

e) $\sin\left(-\frac{25\pi}{6}\right) = -\sin\left(\frac{25\pi}{6}\right) = -\sin\left(4\pi+\frac{\pi}{6}\right) = -\sin\left(\frac{\pi}{6}\right)$

$= -\frac{1}{2}$

$\sin(-x)=-\sin x$ $\quad$ $\tan(-x)=-\tan(x)$

$\cos(-x)=\cos(x)$

f) $\sin\left(\frac{25\pi}{2}\right)+\cos\left(\frac{11\pi}{2}\right)$

$= \sin\left(12\pi+\frac{\pi}{2}\right)+\cos\left(4\pi+\frac{3\pi}{2}\right) = \sin\left(\frac{\pi}{2}\right)+\cos\left(\frac{3\pi}{2}\right)$

$= 1+0 = 1$

g) $\sin(20\pi)+4\cos(9\pi)$

$= 0+4(-1) = -4$

$\sin(n\pi)=0$

$\cos(n\pi)=1$ even

$\cos(n\pi)=-1$ odd

h) $2\cos(61\pi)+\sin\left(\frac{19\pi}{2}\right)$

$= 2(-1)+\sin\left(8\pi+\frac{3\pi}{2}\right) = -2+\sin\left(\frac{3\pi}{2}\right)$

$= -2-1 = -3$

(i) $5\tan\left(\frac{81\pi}{4}\right)-8\cot\left(-\frac{63\pi}{4}\right)$

$= 5\tan\left(20\pi+\frac{\pi}{4}\right)+8\cot\left(\frac{63\pi}{4}\right)$

$= 5\tan\left(20\pi+\frac{\pi}{4}\right)+8\cot\left(15\pi+\frac{3\pi}{4}\right)$

$= 5\tan\left(\frac{\pi}{4}\right)+8\cot\left(\frac{3\pi}{4}\right)$

$= 5(1)+8(-1) = -3$

Restrictions!!!

Know inverse trig values! These are very quick questions if you understand the notion of "inverse trig functions".

2. Evaluate the following:

a) $\arcsin\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$ $\sin\alpha = \frac{\sqrt{3}}{2}$ α is in QI or QIV

b) $\arccos\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$ $\cos\alpha = -\frac{1}{2}$ α is in QI or QII

c) $\arctan(-1) = -\frac{\pi}{4}$ $\tan\alpha = -1$ α is in QI or QIV

d) $\arcsin\left(-\frac{1}{2}\right) + \arctan(1) + \arccos\left(\frac{1}{2}\right)$

$$= -\frac{\pi}{6} + \frac{\pi}{4} + \frac{\pi}{3} = \frac{-2\pi + 3\pi + 4\pi}{12} = \frac{5\pi}{12}$$

e) $\arcsin\left(\frac{1}{2}\right) + \arccos\left(-\frac{\sqrt{2}}{2}\right) + \arctan(-1)$

$$= \frac{\pi}{6} + \frac{3\pi}{4} - \frac{\pi}{4} = \frac{\pi}{6} + \frac{2\pi}{4} = \frac{\pi}{6} + \frac{\pi \cdot 3}{2 \cdot 3} = \frac{\pi + 3\pi}{6} = \frac{4\pi}{6} = \frac{2\pi}{3}$$

f) $\arctan(-1) + \arcsin\left(-\frac{1}{2}\right) + \arccos\left(\frac{\sqrt{2}}{2}\right)$

$$= \cancel{-\frac{\pi}{4}} - \frac{\pi}{6} + \cancel{\frac{\pi}{4}} = -\frac{\pi}{6}$$

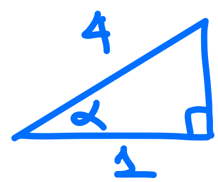
3. Find the exact value of the following. If undefined, state, *undefined*.

a) $\sin\left(\overset{\alpha}{\arccos\left(\frac{1}{4}\right)}\right) = \sin(\alpha) = \frac{\sqrt{15}}{4}$

$\alpha = \arccos\left(\frac{1}{4}\right)$

$\cos(\alpha) = \frac{1}{4} = \frac{adj}{hyp}$

QI



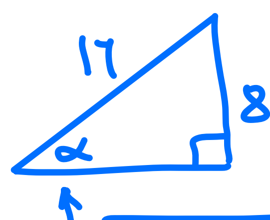
$\sqrt{4^2 - 1^2} = \sqrt{15}$

b) $\tan\left(\overset{\alpha}{\arcsin\left(\frac{8}{17}\right)}\right) = \tan\alpha = \frac{8}{15}$

$\alpha = \arcsin\left(\frac{8}{17}\right)$

$\sin\alpha = \left(\frac{8}{17}\right) = \frac{opp}{hyp}$

QI



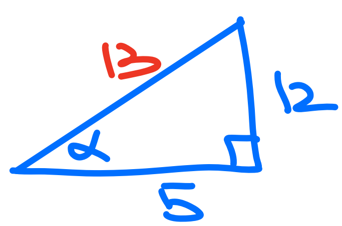
$\sqrt{17^2 - 8^2} = 15$

c) $\csc\left(\overset{\alpha}{\arctan\left(\frac{12}{5}\right)}\right) = \csc(\alpha) = \frac{13}{12}$

$\alpha = \arctan\left(\frac{12}{5}\right)$

$\tan\alpha = \frac{12}{5} = \frac{opp}{adj}$

QI



5-12-13

$A\cos(Bx-C)+D$ $\quad$ $A\sin(Bx-C)+D$

$2\pi/B$

$A\tan(Bx-C)+D$ $\quad$ $A\cot(Bx-C)+D$

π/B

4. Find the periods of the following functions:

a) $f(x)=2\sin(4x-1)$ $\quad p = 2\pi/4 = \pi/2$

b) $g(x)=-4\cos\left(\frac{x}{2}+1\right)+5$ $\quad p = 2\pi \div \frac{1}{2} = 2\pi \cdot 2 = 4\pi$

c) $f(x)=2\tan(\pi x-1)+19$ $\quad p = \pi/\pi = 1$

d) $f(x)=2\sin(10\pi x+4)+5$ $\quad p = \frac{2\pi}{10\pi} = \frac{1}{5}$

5. Give an equation of the form $f(x) = A\cos(Bx - C) + D$ which satisfies the following description.
"The amplitude is 3, the phase shift is 2 to the left, vertical shift is 4, and the period is $\frac{\pi}{3}$."

~~a) $f(x) = 3\cos(6\pi x - 12) + 4$~~
b) $f(x) = 3\cos(6x + 12) + 4$
~~c) $f(x) = 6\cos(3\pi x - 12) + 4$~~
d) $f(x) = 3\cos(6x - 12) + 4$
~~e) $f(x) = 6\cos(6x + 2) + 4$~~

$\frac{2\pi}{B} \times \frac{\pi}{3}$

$6\pi = B\pi \quad B = 6$

$\frac{C}{B} = -2$

$\frac{C}{6} = -2$

$C = -12$

$6x - (-12)$

$6x + 12$

6. Give an equation of the form $f(x) = A\tan(Bx - C) + D$ that could represent the following graph.

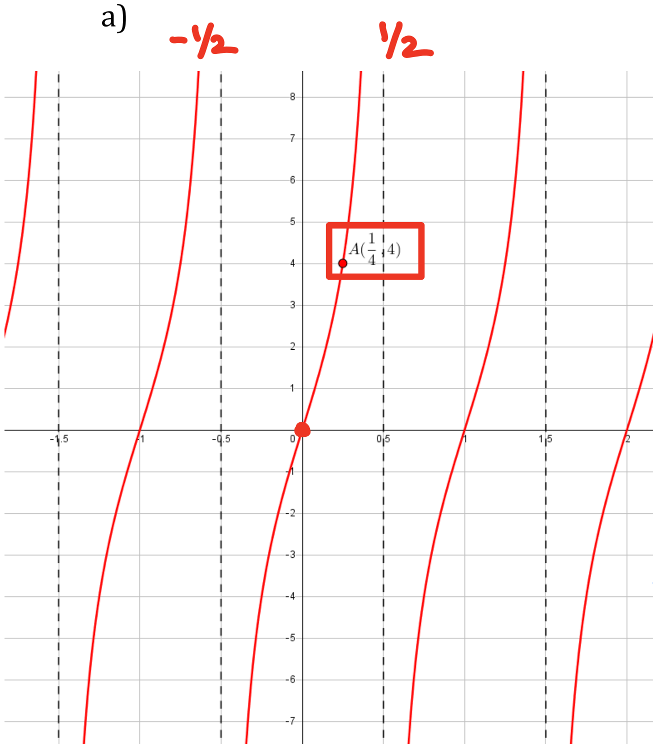
period $= \frac{1}{2} - (-\frac{1}{2}) = 1$

$\frac{\pi}{B} = 1 \Rightarrow B = \pi$

no phase shift $\Rightarrow C = 0$

no vertical shift $\Rightarrow D = 0$

a)



$f(x) = A\tan(\pi x)$

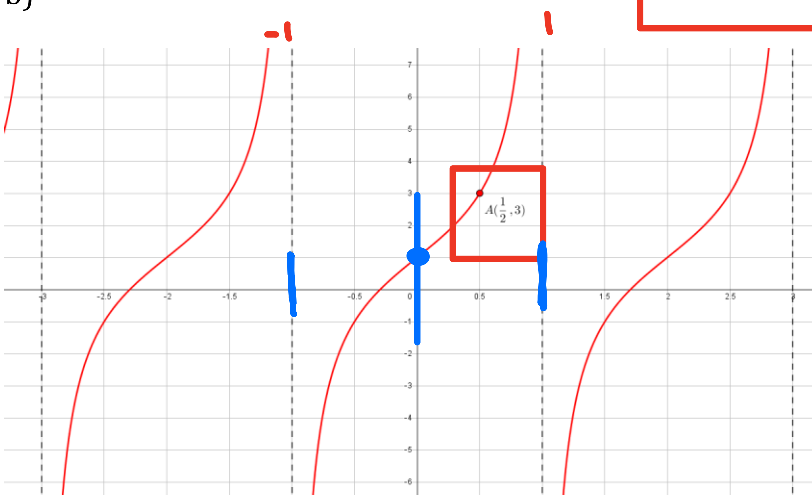
$A(\frac{1}{4}, 4)$ $\quad f(\frac{1}{4}) = 4$

$f(\frac{1}{4}) = A\tan(\pi \cdot \frac{1}{4}) = 4$

$A(1) = 4 \Rightarrow A = 4$

$$\boxed{f(x) = 4\tan(\pi x)}$$

b)



period $= 2$

$\frac{\pi}{B} \cancel{\times} \frac{2}{1}$ $\quad 2B = \pi$

$B = \frac{\pi}{2}$

no phase shift $\Rightarrow C = 0$

$D = 1$

$f(x) = A\tan(\frac{\pi}{2}x) + 1$

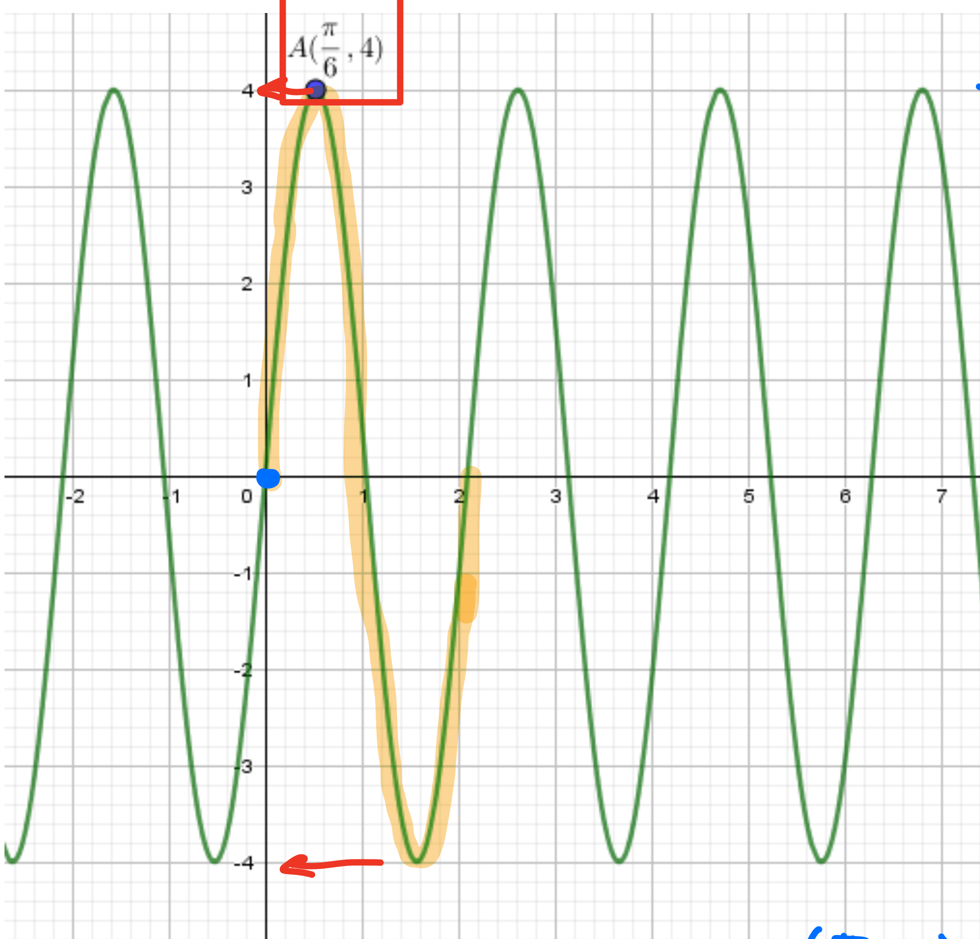
$A(\frac{1}{2}, 3)$ $\quad f(\frac{1}{2}) = A\tan(\frac{\pi}{2} \cdot \frac{1}{2}) + 1 = 3$

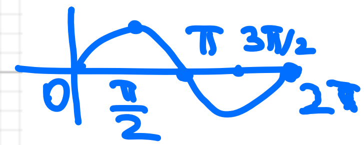
$f(\frac{1}{2}) = 3$

$A(1) + 1 = 3$ $\quad A = 2$

$$\boxed{f(x) = 2\tan\left(\frac{\pi}{2}x\right) + 1}$$

7. For the following graph give an equation of the form $f(x) = A\sin(Bx - C) + D$ which could be used to represent the graph.





no phase shift $C=0$

no vertical shift $D=0$

$f(x) = A\sin(Bx)$

amp. $= 4$

$A = 4$

$\frac{2\pi}{B}$

$f(x) = 4\sin(Bx)$ $\quad A\left(\frac{\pi}{6}, 4\right)$

$f\left(\frac{\pi}{6}\right) = 4\sin\left(B\cdot\frac{\pi}{6}\right) = 4$

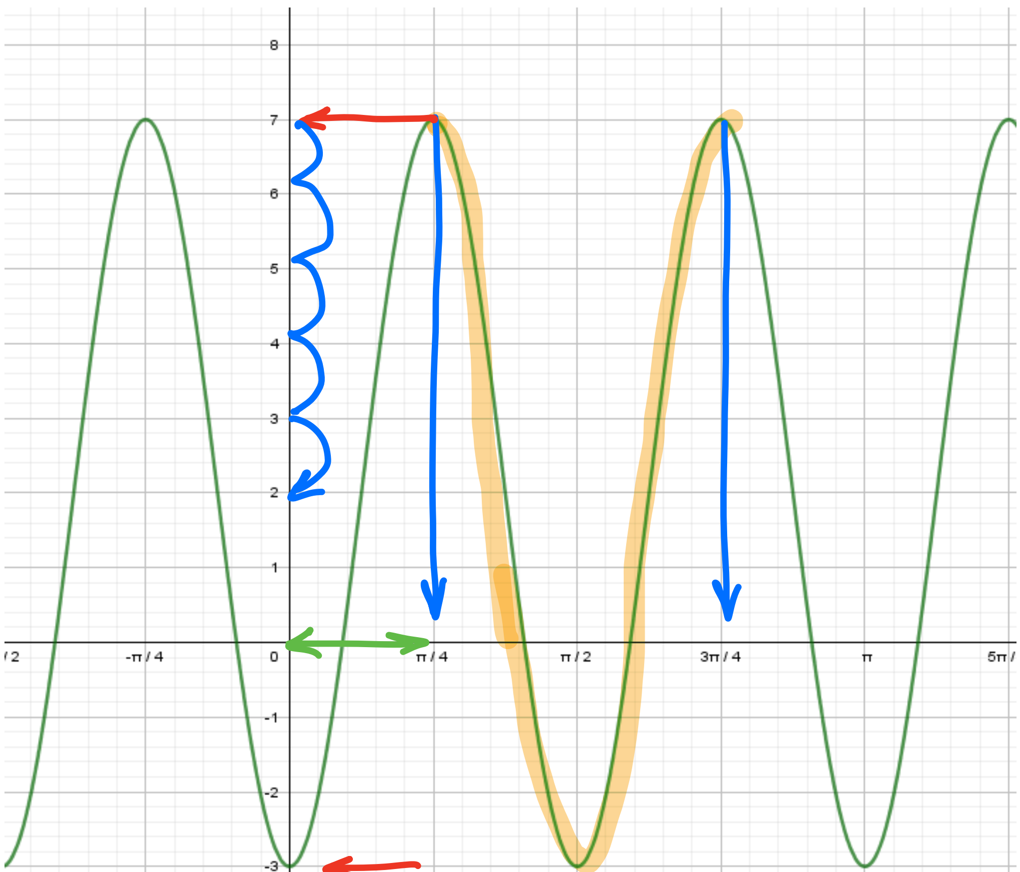
$\sin\left(B\cdot\frac{\pi}{6}\right) = 1$

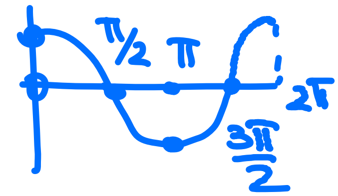
$B\cdot\frac{\cancel{\pi}}{6} = \frac{\cancel{\pi}}{2}$ $\quad \frac{B}{6} = \frac{1}{2}$ $\quad \Rightarrow 2B = 6$

$B = 3$

$$\boxed{f(x) = 4\sin(3x)}$$

8. For the following graph give an equation of the form $f(x) = A\cos(Bx - C) + D$ which could be used to represent the graph.





period $= \frac{3\pi}{4} - \frac{\pi}{4} = \frac{2\pi}{4} = \frac{\pi}{2}$ $\qquad \frac{2\pi}{B} = \frac{\pi}{2}$

$4\pi = B\pi \Rightarrow B = 4$

amplitude : $\frac{\max - \min}{2} = \frac{7-(-3)}{2} = 5 = A$

$D = 2$

phase shift : $\frac{C}{B} = \frac{\pi}{4}$ $\qquad \frac{C}{4} = \frac{\pi}{4} \Rightarrow C = \pi$

$$\boxed{f(x) = 5\cos(4x - \pi) + 2}$$

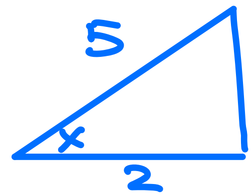
QI

9. Suppose that $\cos(x) = \frac{2}{5}$ and $0 < x < \frac{\pi}{2}$. Find the values of:

a) $\cos(2x) = 2\cos^2(x) - 1$

$$= 2\left(\frac{2}{5}\right)^2 - 1 = 2\left(\frac{4}{25}\right) - 1$$

$$= \frac{8}{25} - \frac{25}{25} = \boxed{-\frac{17}{25}}$$



$\sqrt{5^2 - 2^2} = \sqrt{21}$

$\cos x = \frac{2}{5} = \frac{\text{adj}}{\text{hyp}}$

b) $\sin(2x) = 2 \overset{?}{\sin} x \overset{\checkmark}{\cos} x$

$$= 2\left(\frac{\sqrt{21}}{5}\right)\left(\frac{2}{5}\right) = \boxed{\frac{4\sqrt{21}}{25}}$$

QI

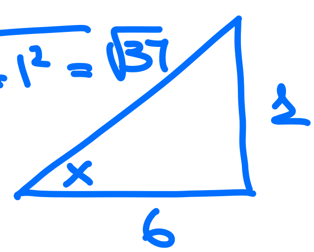
10. Given: $\cot(x) = 6$ and $0 < x < \frac{\pi}{2}$, find the value of $\sin(2x)$.

$\sin(2x) = 2\sin x \cos x$

$\cot(x) = \frac{6}{1} = \frac{\text{adj}}{\text{opp}}$

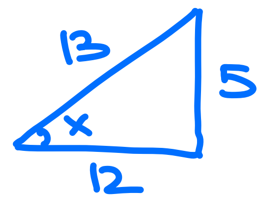
$$\sin(2x) = 2\left(\frac{1}{\sqrt{37}}\right)\left(\frac{6}{\sqrt{37}}\right)$$

$\sqrt{6^2 + 1^2} = \sqrt{37}$



$$= \boxed{\frac{12}{37}}$$

$5-12-13$



QI

11. Given: $\sin(x)=\dfrac{5}{13}$ and $0<x<\dfrac{\pi}{2}$, find the value of the following expressions using sum and difference formulas:

a) $\sin\left(\underset{A}{x}+\underset{B}{\dfrac{\pi}{4}}\right) = \sin x \cos\frac{\pi}{4} + \cos x \sin\frac{\pi}{4}$

$$= \frac{5}{13}\left(\frac{\sqrt{2}}{2}\right) + \frac{12}{13}\left(\frac{\sqrt{2}}{2}\right) = \frac{5\sqrt{2}+12\sqrt{2}}{26} = \boxed{\frac{17\sqrt{2}}{26}}$$

b) $\cos\left(x-\dfrac{\pi}{3}\right) = \cos x \cos\frac{\pi}{3} + \sin x \sin\frac{\pi}{3}$

$$= \frac{12}{13}\left(\frac{1}{2}\right) + \left(\frac{5}{13}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{12+5\sqrt{3}}{26}$$

c) $\tan\left(x+\dfrac{3\pi}{4}\right) = \dfrac{\tan x + \tan\left(\frac{3\pi}{4}\right)}{1-\tan x \tan\left(\frac{3\pi}{4}\right)}$

$\tan\left(\frac{3\pi}{4}\right) = -1$

$\tan x = \frac{5}{12}$

$$= \frac{\frac{5}{12}+(-1)}{1-\frac{5}{12}(-1)} = \frac{\frac{5}{12}-\frac{12}{12}}{\frac{12}{12}+\frac{5}{12}}$$

$$= \frac{-\frac{7}{12}}{\frac{17}{12}} = -\frac{7}{12} \div \frac{17}{12} = -\frac{7}{12}\cdot\frac{12}{17}$$

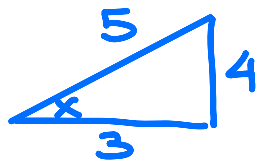
$$= \boxed{-\frac{7}{17}}$$

QI

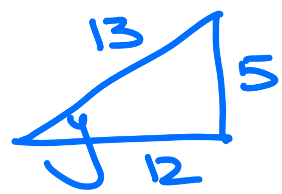
12. Given: $\sin(x)=\frac{4}{5}$, $\cos(y)=\frac{12}{13}$ and $0<x,y<\frac{\pi}{2}$, find the value of the following expressions using sum and difference formulas: $\sin(x-y)$

$= \sin x \cos y - \cos x \sin y$

For x:



For y:



3-4-5

5-12-13

$$= \left(\frac{4}{5}\right)\left(\frac{12}{13}\right) - \left(\frac{3}{5}\right)\left(\frac{5}{13}\right) = \frac{48-15}{65} = \boxed{\frac{33}{65}}$$

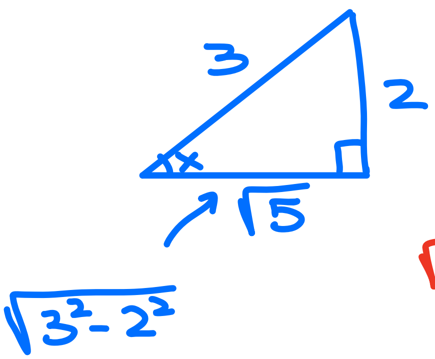
QI

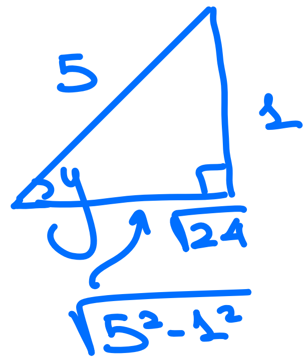
13. Given: $\sin(x)=\frac{2}{3}$, $\sin(y)=\frac{1}{5}$ and $0<x,y<\frac{\pi}{2}$, find the value of the following expressions using sum and difference formulas: $\cos(x+y)$

$= \cos x \cos y - \sin x \sin y$

For x: $\sin x = \frac{2}{3} = \frac{\text{opp}}{\text{hyp}}$

For y: $\sin y = \frac{1}{5} = \frac{\text{opp}}{\text{hyp}}$





$\sqrt{4 \cdot 6} = 2\sqrt{6}$

$= \left(\frac{\sqrt{5}}{3}\right)\left(\frac{\sqrt{24}}{5}\right) - \left(\frac{2}{3}\right)\left(\frac{1}{5}\right)$

$= \left(\frac{\sqrt{5}}{3}\right)\left(\frac{2\sqrt{6}}{5}\right) - \left(\frac{2}{3}\right)\left(\frac{1}{5}\right) = \frac{2\sqrt{30}}{15} - \frac{2}{15}$

$= \boxed{\frac{2\sqrt{30} - 2}{15}}$

$\frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}$

$\cos^2 x + \sin^2 x = 1$

$\cos^2 x - \sin^2 x = \cos(2x)$ $\quad \sin(2x) = 2\sin x \cos x$

14. Simplify the following expressions:

a) $\dfrac{40\sin(x)\cos(x)}{\cos^2(x)-\sin^2(x)} = \dfrac{20 \cdot 2\sin x \cos x}{\cos(2x)}$

$= \dfrac{20\sin(2x)}{\cos(2x)} = \boxed{20\tan(2x)}$

$\cot(-x) = -\cot(x)$

$\tan(x)\cot(x) = 1$

b) $5\sin^2(x)+5\cos^2(x)-7\tan(x)\cot(-x)$

$= 5(\sin^2(x)+\cos^2(x)) - 7\tan(x)\cdot(-\cot(x))$

$= 5(1) + 7(1) = \boxed{12}$

$\dfrac{\sin(x)}{\cos(x)} \cdot \dfrac{\cos(x)}{\sin(x)} = 1$

c) $60\sin(-t)\cos(-t)$

$\sin(-t) = -\sin(t)$

$\cos(-t) = \cos(t)$

$= -60\sin(t)\cos(t)$

$= -30 \cdot 2\sin(t)\cos(t) = \boxed{-30\sin(2t)}$

d) $\cos(-t)+\cos(-t)\tan^2(-t)$

ignore

$a + a \cdot b^2 = a[1+b^2]$

$= \cos t + \cos t \cdot \tan^2(t) = \cos t[1+\tan^2 t]$

$= \cos t \cdot \sec^2 t = \cancel{\cos t} \cdot \dfrac{1}{\cos^{\cancel{2}} t}$

$\tan(-t) = -\tan(t)$

$1+\tan^2 t = \sec^2 t$

$= \dfrac{1}{\cos t}$

$= \boxed{\sec t}$

$[\tan(-t)]^2 = \tan(-t)\cdot\tan(-t)$

$= -\tan(t)\cdot(-\tan(t)) = \tan^2(t)$

QI

15. Given $\cos(t) = \frac{4}{5}$ and $0 < t < \frac{\pi}{2}$, find the value of the following expression:

$\sin(-t) + \cos(-t)\tan^2(-t)$ $= -\sin(t) + \cos(t)\tan^2(t)$

a) $-\frac{21}{20}$

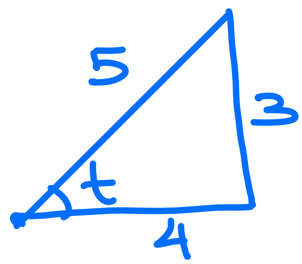
b) $-\frac{9}{4}$

c) $\frac{3}{20}$

d) $\frac{3}{5}$

e) $-\frac{3}{20}$

f) None of the above



$$= -\frac{3}{5} + \frac{4}{5}\left(\frac{3}{4}\right)^2$$

$$= -\frac{3}{5} + \frac{\cancel{4}}{5} \cdot \frac{9}{\cancel{16}\,4}$$

$$= -\frac{3 \cdot 4}{5 \cdot 4} + \frac{9}{20} = \frac{-12 + 9}{20}$$

$$= \boxed{\frac{-3}{20}}$$

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$$

16. State whether the following equations are TRUE or FALSE:

F $\sin(-x) = \sin(x)$ $\sin(-x) = -\sin x$

T $\cos(-x) = \cos(x)$

T $1 + \tan^2(t) = \sec^2(t)$

F $\sin(2x) = 2\sin(x)$ $\sin(2x) = 2\sin x \cos x$

T $\tan(-x) = -\tan(x)$

F $\cos(2x) = 2\cos(x) - 1$ $\cos(2x) = 2\cos^2 x - 1$

T $\sin^2(x) = 1 - \cos^2(x)$

F $\sin(x + \pi) = \sin(x)$ $\sin(x + 2\pi) = \sin(x)$

T $\cos(x + 20\pi) = \cos(x)$

T $\tan(x + 45\pi) = \tan(x)$

T $\tan(x + 44\pi) = \tan(x)$

Exercise:

Graph $f(x) = 6\sin(4x)$ for one period.

a) State the period.
b) Label the x and y intercepts (if any) with ordered pairs.
c) Label the maximum value(s) (if any) with an ordered pair.
d) Label the minimum value(s) (if any) with an ordered pair.
e) State the domain of the function.
f) State the range of the function.