MATH 2313 Test 4

Students are responsible for reserving a seat for the test using **Schedule Exams** tab on CASA BEFORE the first day of test while seats are available. You take your test in a **CASA Testing Center**.

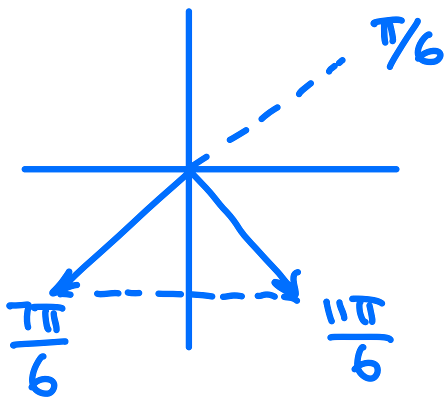
If you miss your scheduled test, then check the scheduler to see if there are other available times and if you can reschedule it. Your instructor does not control the scheduler, cannot add test slots, and/or reschedule your test – make sure you do not miss your reserved time.

FAQ

- ✓ **What is covered on the test?** 6.3 Chapter 7, Vectors, Chapter 8
- ✓ **How many questions are there?** About 20 Multiple-choice questions to be completed in 60 minutes.
- ✓ **What calculator can I use on exams?** No calculators or other software are allowed on exams – study accordingly.
- ✓ **Will there be a formula sheet provided?** No, not on Test 4. You canNOT have a copy of a unit circle while taking the test – make sure you KNOW YOUR UNIT CIRCLE!
- ✓ **How do I study for the best result?**
  1. Begin with this review. SOLVE a blank copy of this review after it is discussed in class!
     This review should not be your only source while studying for the exam. It covers most of the "skills" you should be practicing, but it is not intended to be the only source.
  2. Take Practice Test 4 BEFORE your test! It is for practice AND extra credit. 5% of your best score will be added to your Test 4 score. Retake Practice Test 4 several times to strengthen your skills.
  3. Review online quizzes and homework.

6.3

1. Solve the following equation over the interval $[0, 2\pi)$: one period



$$2\sin(x) - 3 = -4$$

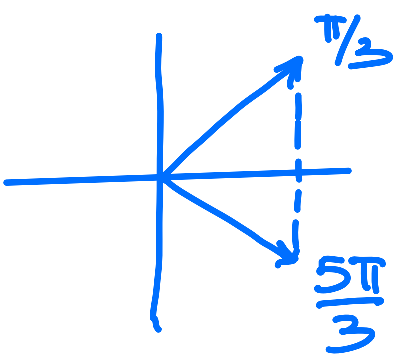
$$+3 \quad +3$$

$$2\sin(x) = -1$$

$$\sin(x) = -\frac{1}{2}$$

$$\boxed{x = \frac{7\pi}{6} \qquad x = \frac{11\pi}{6}}$$

2. Solve the following equation over the interval $[0, 2\pi)$: one period



$$8\cos(x) - 5 = -1$$

$$+5 \quad +5$$

$$8\cos(x) = 4$$

$$\cos(x) = \frac{4}{8} = \frac{1}{2}$$

$$\boxed{x = \frac{\pi}{3} \qquad x = \frac{5\pi}{3}}$$

$A\sin(Bx - C) + D$ period $= \frac{2\pi}{B}$ $p = \frac{2\pi}{2} = \pi$

3. Solve the following equation over the interval $[0, \pi)$: one period

$$4\sin(2x) = 2\sqrt{2}$$

$$\sin(2x) = \frac{2\sqrt{2}}{4}$$

$$\sin(2x) = \frac{\sqrt{2}}{2}$$

$2x = \frac{\pi}{4}$ $2x = \frac{3\pi}{4}$

$\boxed{x = \frac{\pi}{8} \qquad x = \frac{3\pi}{8}}$

4. Solve the following equation over the interval $[0, \frac{\pi}{2})$: one period

$$6\sin(4x) = -3\sqrt{3}$$

new period $= \frac{2\pi}{B}$

$= \frac{2\pi}{4}$

$= \frac{\pi}{2}$

$$\sin(4x) = -\frac{3\sqrt{3}}{6} = -\frac{\sqrt{3}}{2}$$

$\frac{4x}{4} = \frac{\cancel{4}\pi}{3} \cdot \cancel{4}$ $\frac{4x}{4} = \frac{5\pi}{3} \cdot \frac{1}{4}$

$\boxed{x = \frac{\pi}{3} \qquad x = \frac{5\pi}{12}}$

5. Solve the following equation over the interval $[0, \frac{\pi}{2})$: one period

$$8\sin^2(4x) - 1 = 1$$

$+1 \quad +1$

$8\sin^2(4x) = 2$

$\sin^2(4x) = \frac{2}{8} = \frac{1}{4}$

$\sqrt{\sin^2(4x)} = \pm\sqrt{\frac{1}{4}}$

$\sin(4x) = \pm\frac{1}{2}$

① $\sin(4x) = \frac{1}{2}$

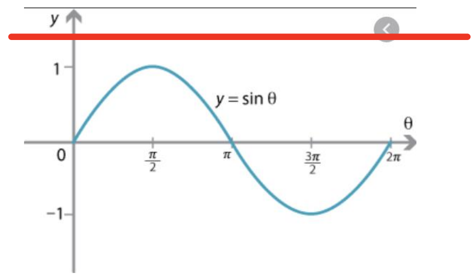
$4x = \frac{\pi}{6}$ or $4x = \frac{5\pi}{6}$

② $\sin(4x) = -\frac{1}{2}$

$4x = \frac{7\pi}{6}$ or $4x = \frac{11\pi}{6}$

$$\boxed{x = \frac{\pi}{24} \text{ or } x = \frac{5\pi}{24} \qquad x = \frac{7\pi}{24} \text{ or } x = \frac{11\pi}{24}}$$

6. How many solutions are there to the following equations? (Hint: You can answer this without solving the equations!)

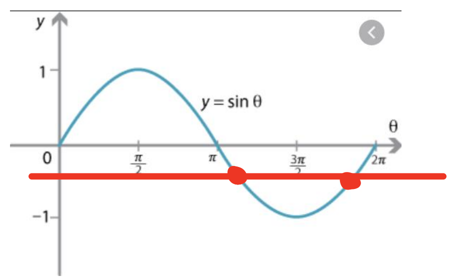


$y = \frac{5}{4}$

a) $4\sin(x) = 5$ over the interval $[0, 2\pi)$.

$\sin(x) = \frac{5}{4} > 1$

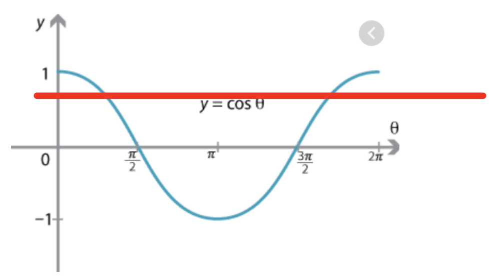
0 solutions



b) $5\sin(x) = -1$ over the interval $[0, 2\pi)$.

$\sin(x) = -\frac{1}{5}$

2 solutions



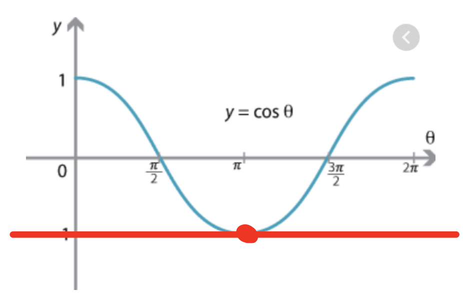
c) $5\cos(x) + 1 = 4$ over the interval $[0, 2\pi)$.

$-1 \quad -1$

$5\cos(x) = 3$

$\cos(x) = \frac{3}{5}$

2 solutions

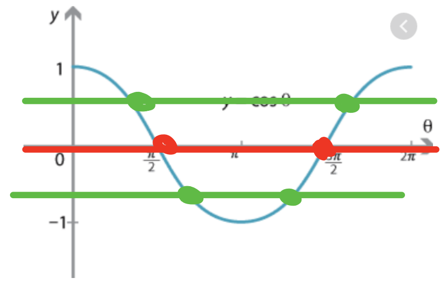


d) $2\cos(x) + 2 = 0$ over the interval $[0, 2\pi)$.

$2\cos(x) = -2$

$\cos(x) = -1$

1 solution



e) $2\cos(x) + 4 = 4$ over the interval $[0, 2\pi)$.

$2\cos(x) = 0$ $\cos(x) = 0$ 2 solutions

f) $2\cos^2(x) + 1 = 2$ over the interval $[0, 2\pi)$.

$2\cos^2(x) = 1$

$\sqrt{\cos^2(x)} = \pm\sqrt{\frac{1}{2}}$

$\cos x = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\cos x = -\frac{\sqrt{2}}{2}$

4 solutions

g) $2\cos^2(2x) + 1 = 2$ over the interval $[0, 2\pi)$.

$\cos(2x) = \pm\frac{\sqrt{2}}{2}$

period of $\cos(2x)$
$= \frac{2\pi}{2} = \pi$

8 solution

h) $4\sin^2(5x) + 2 = 3$ over the interval $[0, 2\pi)$.

$-2 \quad -2$

$4\sin^2(5x) = 1$

$\sqrt{\sin^2(5x)} = \pm\sqrt{\frac{1}{4}}$

$\sin(5x) = \frac{1}{2}$

$\sin(5x) = -\frac{1}{2}$

$\sin(x) = \frac{1}{2}$
2 sol.
$\sin(x) = -\frac{1}{2}$
2 sol.

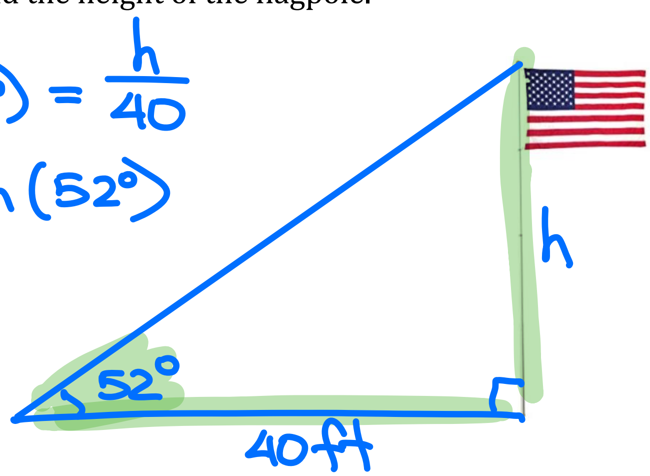
20 solutions

7. The angle of elevation to the top of a flag pole from a point on the ground 40 feet from the base of the pole is 52°. Find the height of the flagpole.

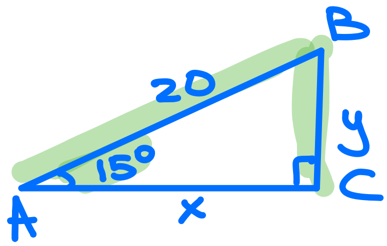
a) $40\sin(52^\circ)$
b) $40\cos(52^\circ)$
c) $40\sec(52^\circ)$
d) $40\tan(52^\circ)$
e) $40\cot(52^\circ)$

$\tan(52^\circ) = \frac{h}{40}$

$h = 40\tan(52^\circ)$



8. In a right triangle ABC with right angle C, angle A measures 15°. If the hypotenuse is 20 units long, find the lengths of the legs AC and BC.



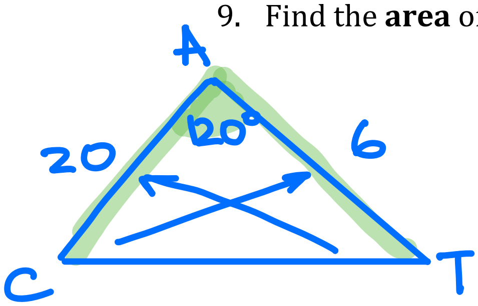
$$\cos(15^{\circ}) = \frac{x}{20}$$

$$\boxed{x = 20\cos(15^{\circ})}$$

$$\sin(15^{\circ}) = \frac{y}{20}$$

$$\boxed{y = 20\sin(15^{\circ})}$$

9. Find the **area** of triangle CAT if $m\angle A = 120^{\circ}$, $c = 6, t = 20$.

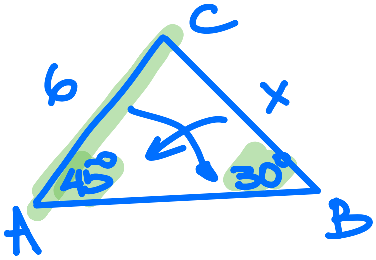


$$A = \frac{1}{2}\,\text{side} \cdot \text{side} \cdot \sin(A)$$

$$= \frac{1}{\cancel{2}} \cdot \overset{3}{\cancel{6}} \cdot 20 \cdot \sin(120^{\circ})$$

$$= \overset{30}{\cancel{60}} \cdot \frac{\sqrt{3}}{\cancel{2}} = \boxed{30\sqrt{3}}$$

10. In triangle ABC, $m\angle A = 45^{\circ}$, $m\angle B = 30^{\circ}$ and $AC = 6$. Find the length of BC.



LAW of SINES

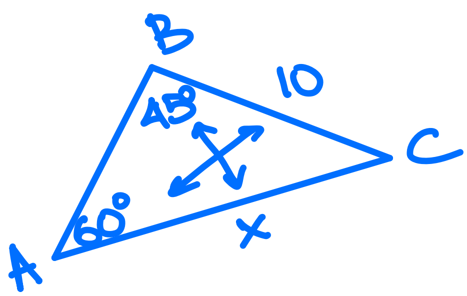
$$\frac{x}{\sin 45^{\circ}} = \frac{6}{\sin 30^{\circ}}$$

$$x \sin 30^{\circ} = 6 \sin 45^{\circ}$$

$$x \cdot \frac{1}{2} = 6 \frac{\sqrt{2}}{2}$$

$$\boxed{x = 6\sqrt{2}}$$

11. In triangle ABC, $m\angle A = 60^{\circ}$, $m\angle B = 45^{\circ}$ and $BC = 10$. Find the length of AC.



$$\frac{x}{\sin 45^{\circ}} = \frac{10}{\sin 60^{\circ}}$$

$$x \sin 60^{\circ} = 10 \sin 45^{\circ}$$

$$x \frac{\sqrt{3}}{2} = 10 \frac{\sqrt{2}}{2}$$

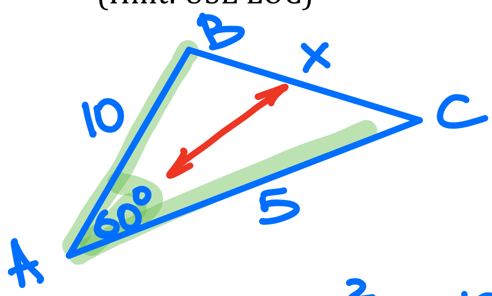
$$x\sqrt{3} = 10\sqrt{2}$$

$$x = \frac{10\sqrt{2} \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} = \boxed{\frac{10\sqrt{6}}{3}}$$

12. In triangle ABC, $m\angle A = 60^{\circ}$, $AB = 10$ and $AC = 5$. Find the length of BC.

(Hint: USE LOC)

SAS → LOC



$$x^2 = 10^2 + 5^2 - 2(5)(10)\cos(60°)$$

$$x^2 = 100 + 25 - \cancel{2} \cdot 50 \cdot \frac{1}{\cancel{2}}$$

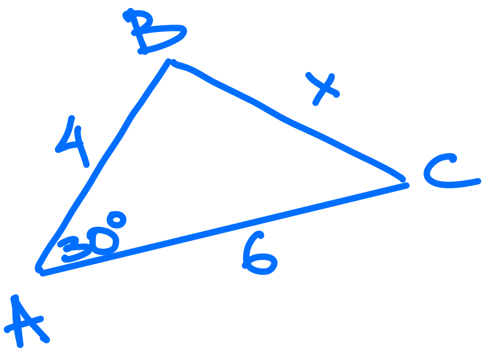
$$\sqrt{x^2} = \sqrt{75}$$

$$x = \sqrt{75} = \sqrt{25 \cdot 3} = \sqrt{25} \cdot \sqrt{3} = \boxed{5\sqrt{3}}$$

13. In triangle ABC, $m\angle A = 30^{\circ}$, $AB = 4$ and $AC = 6$. Find the length of BC.

(Hint: USE LOC)

SAS → LOC



$$x^2 = 6^2 + 4^2 - 2(6)(4)\cos(30^{\circ})$$

$$x^2 = 36 + 16 - \cancel{2} \cdot 24 \cdot \frac{\sqrt{3}}{\cancel{2}}$$

$$\sqrt{x^2} = \sqrt{52 - 24\sqrt{3}}$$

$$x = \sqrt{52 - 24\sqrt{3}} = \sqrt{4(13 - 6\sqrt{3})} = \sqrt{4}\sqrt{13 - 6\sqrt{3}}$$

$$= \boxed{2\sqrt{13 - 6\sqrt{3}}}$$

$\vec{v} = a\vec{i} + b\vec{j}$ $\quad \|\vec{v}\| = \sqrt{a^2 + b^2}$

14. Let $\vec{u} = 4\vec{\imath} + 3\vec{\jmath}$ and $\vec{v} = \vec{\imath} - 2\vec{\jmath}$

a) Find magnitude of vector $\vec{u}$.

$\|\vec{u}\| = \sqrt{4^2 + 3^2} = 5$

$\|\vec{v}\| = \sqrt{1^2 + (-2)^2} = \sqrt{5}$

b) Find the vector $5\vec{u} - 2\vec{v}$.

$\vec{u} = \langle 4, 3 \rangle$

$\vec{v} = \langle 1, -2 \rangle$

$$\begin{array}{rl} & 5\vec{u} = \langle 20, 15 \rangle \\ - & 2\vec{v} = \langle \overset{-}{2}, \overset{-}{-4} \rangle \\ \hline & \langle 18, 19 \rangle \end{array}$$

$5\vec{u} - 2\vec{v} = 18\vec{i} + 19\vec{j}$

15. Let $\vec{u} = \langle 2, -1 \rangle$ and $\vec{v} = \langle 5, 3 \rangle$.

a) Find the vector $2\vec{u} + 10\vec{v}$.

$\vec{u} = \langle 2, -1 \rangle$

$\vec{v} = \langle 5, 3 \rangle$

$$\begin{array}{rl} & 2\vec{u} = \langle 4, -2 \rangle \\ + & 10\vec{v} = \langle 50, 30 \rangle \\ \hline & \langle 54, 28 \rangle \end{array}$$

$54\vec{i} + 28\vec{j}$

b) Find the dot product of these vectors: $\vec{u} \cdot \vec{v}$

$\vec{u} = \langle a, b \rangle$ $\quad \vec{v} = \langle c, d \rangle$

$\vec{u} \cdot \vec{v} = a \cdot c + bd$

$\langle 2, -1 \rangle \cdot \langle 5, 3 \rangle = 10 - 3 = \boxed{7}$

CHAPTER 8

Know:

- How to identify conic sections.
- How to complete the square to get the center or vertices.
- Circles: center, radius, how to graph.
- Ellipses: center, vertices, 4 key points, how to graph.
- Parabolas: vertex, how to graph.
- Hyperbola: how to identify them.
- System: how to solve systems (algebraically or finding the number of solutions graphically)

16. Classify the following:

Choices: Circle, ellipse, hyperbola, parabola, none of the above.

$$\frac{(x-5)^2}{4}-\frac{(y+1)^2}{9}=1$$ hyperbola

$$\frac{(x-5)^2}{4}+\frac{(y+1)^2}{16}=1$$ ellipse

$$(x-5)^2+(y+4)^2=\overset{r^2}{20}$$ circle

$r=\sqrt{20}=\sqrt{4\cdot 5}=2\sqrt{5}$

parabolas

$$(y+1)^2=8(x+2)$$ right

$$(x+1)^2=-4(y+2)$$ down

$y^2 \rightarrow$ horizontal

$x^2 \rightarrow$ vertical $(y=x^2$ ∪$)$

17. Write the equation of a circle with radius 4 and center (-1, 2).

r h k

$(x-h)^2 + (y-k)^2 = r^2$

$(x+1)^2 + (y-2)^2 = 16$

$x: \left(\frac{-8}{2}\right)^2 = (-4)^2 = 16$

18. Find the center and radius of the following circle:

$y: \left(\frac{4}{2}\right)^2 = (2)^2 = 4$

$$x^2 + y^2 - 8x + 4y + 11 = 0$$

$-11 \quad -11$

$(x^2 - 8x + 16) + (y^2 + 4y + 4) = -11 + 16 + 4$

$(x-4)^2 + (y+2)^2 = 9$

center : $(4, -2)$

radius : $\sqrt{9} = 3$

19. Write the following conic section in standard form; identify it and find the center.

$$4x^2 + 25y^2 - 16x + 50y - 59 = 0$$

$(4x^2 - 16x) + (25y^2 + 50y) = 59$

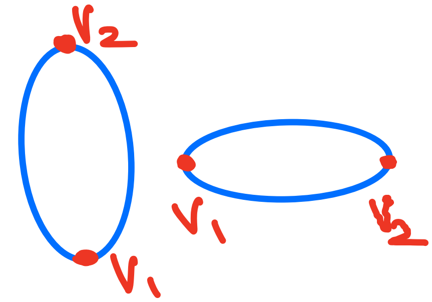
$4(x^2 - 4x + 4) + 25(y^2 + 2y + 1) = 59 + 16 + 25$

$\frac{4(x-2)^2}{100} + \frac{25(y+1)^2}{100} = \frac{100}{100}$

$\frac{(x-2)^2}{25} + \frac{(y+1)^2}{4} = 1$ ellipse

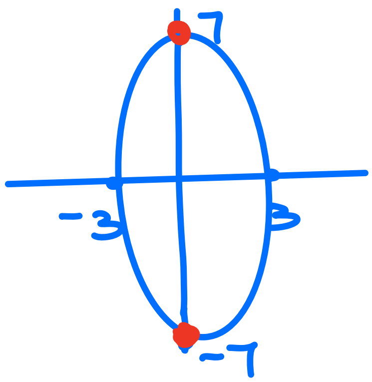
center $(2, -1)$

20. State the vertices of the following ellipses:



a)

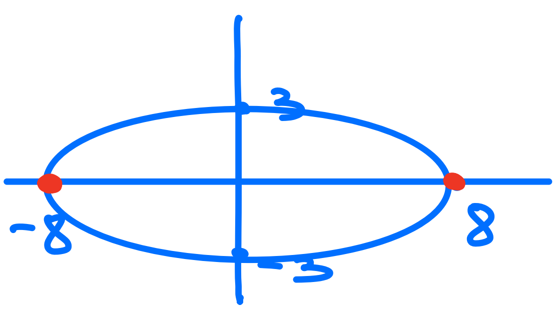
$$\frac{x^2}{9} + \frac{y^2}{49} = 1$$



$$\boxed{(0,7) \ \& \ (0,-7)}$$

b)

$$\frac{x^2}{64} + \frac{y^2}{9} = 1$$



$$\boxed{(-8,0) \ \& \ (8,0)}$$

graph!

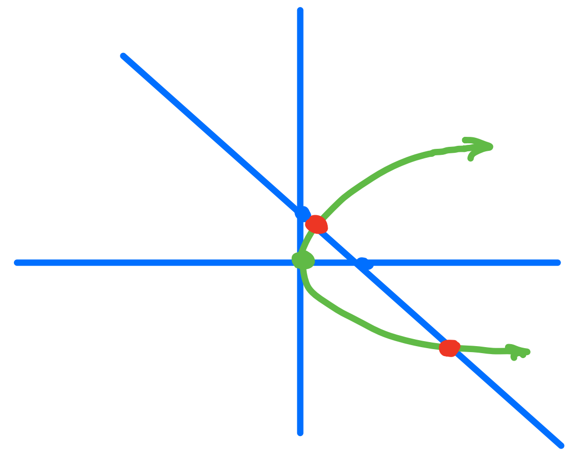
21. How many solutions does the following system have?

line → $y = -x + 1$

parabola → $x = y^2$

x	y
0	1
1	0

2 solutions



22. Find the point(s) of intersection:

$$4x^2 + 7y^2 = 23$$
$$(3x^2 - y^2 = 11)7$$

$$+\begin{array}{r} 4x^2 + \cancel{7y^2} = 23 \\ 21x^2 - \cancel{7y^2} = 77 \\ \hline 25x^2 = 100 \end{array}$$

$\sqrt{x^2} = \pm\sqrt{4}$ $\quad x = \pm 2$

$x = 2$ $\quad 3(2)^2 - y^2 = 11$

$12 - y^2 = 11$ $\quad \sqrt{y^2} = \pm\sqrt{1}$ $\quad y = \pm 1$

$\boxed{(2, 1) \;\&\; (2, -1)}$

$x = -2$ $\quad 3(-2)^2 - y^2 = 11$ $\quad y = \pm 1$

$\boxed{(-2, 1) \quad (-2, -1)}$

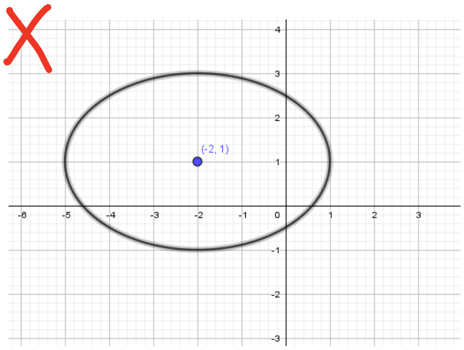
center: (2, -1)

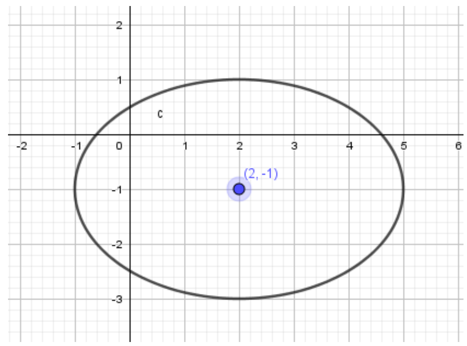
23. Which of the following is the graph of this conic section?

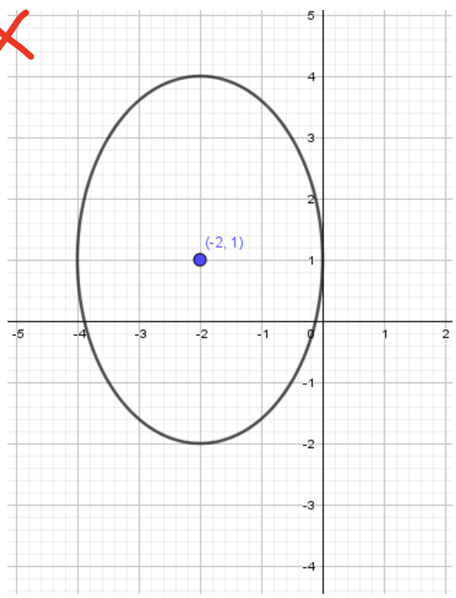
$$\frac{(x-2)^2}{4} + \frac{(y+1)^2}{9} = 1$$

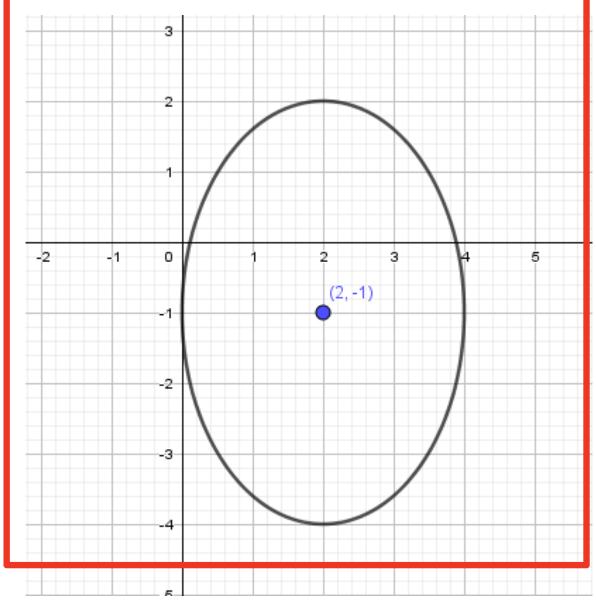
← 2 →

↑ 3 ↓









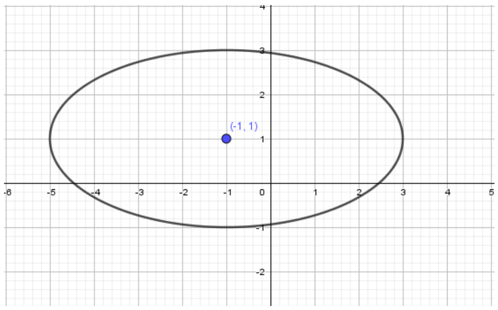
24. Which of the following is the graph of this conic section?

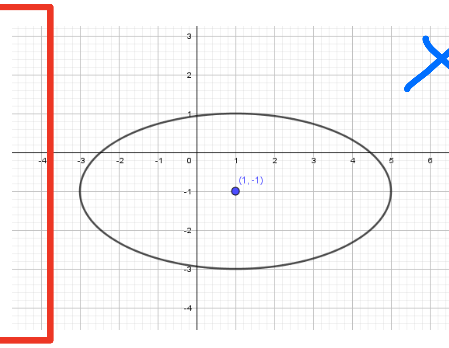
center: (-1, 1)

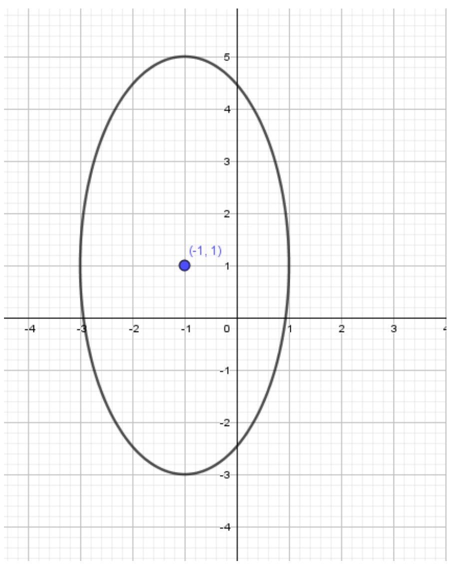
← 4 →

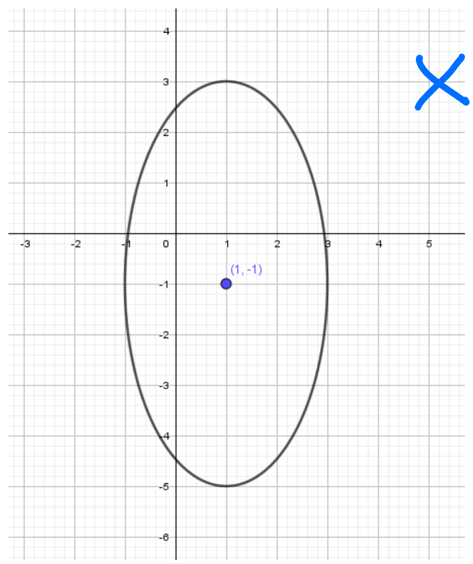
↑ 2 ↓

$$\frac{(x+1)^2}{16}+\frac{(y-1)^2}{4}=1$$









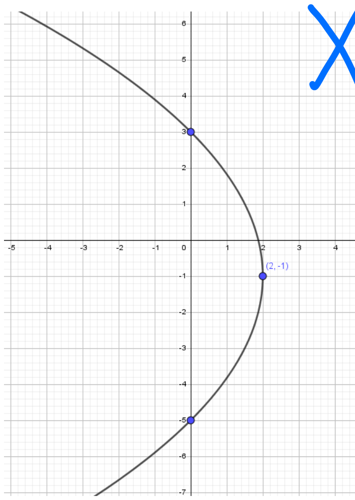
$x, y^2 \to$ horizontal

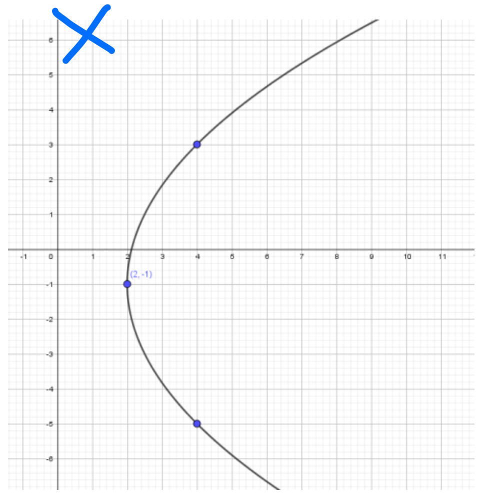
$x^2, y \to$ vertical

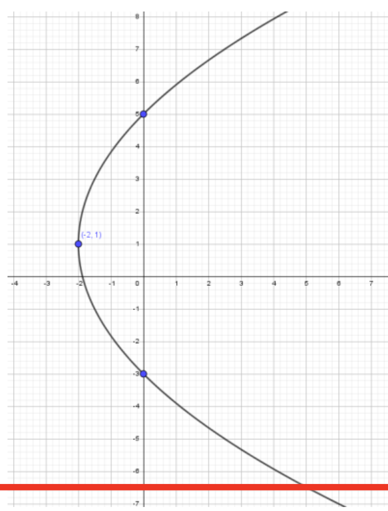
vertex : $(-2, 1)$

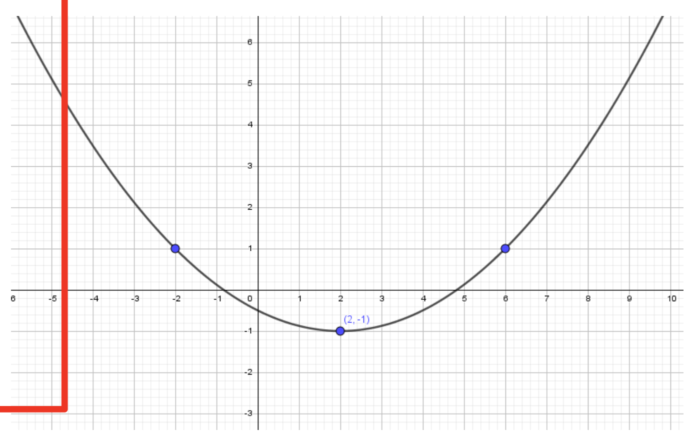
25. Which of the following is the graph of this conic section? $(y-1)^2 = 8(x+2)$

horizontal right









horizontal, left

vertex: (1, -1)

26. Which of the following is the graph of this conic section? $(y+1)^2 = -12(x-1)$