Section 6.3 – Basic Integration Rules

The notation $\int f(x)dx$ is used for an antiderivative of f and called an **indefinite integral**.

$$\int f(x)\,dx = F(x) \text{ means } F'(x) = f(x)$$

In general, to find $\int f(x)dx$, we find an antiderivative of $f(x)$, say $F(x)$, and then we write the indefinite integral as $\int f(x)dx = F(x) + C$. Here, C is called the **constant of integration**.

If given $\int_a^b f(x)dx$, this is a **definite integral** and to evaluate we'll use Part 2 of the

Fundamental Theorem of Calculus: $\int_a^b f(x)dx = F(b) - F(a)$

The Constant Rule for Integrals

$\int k dx = k \cdot x + C$, where k is a constant number.

Example 1: Find of each of the following integrals.

a. $\int 10dx$

b. $\int_1^4 \pi dx$

The Power Rule for Integrals

$\int x^r dx = \frac{x^{r+1}}{r+1} + C$, where $r \neq -1$.

Example 2: Find of each of the following integrals.

a. $\int x^4 dx$

b. $\int_0^1 \frac{1}{\sqrt{x}} dx$

The Constant Multiple of a Function for Integrals

$\int k \cdot f(x) dx = k \int f(x) dx$, where k is a constant number.

Example 3: $\int_{-1}^{1} 4x^7 dx$

The Sum/Difference of Functions for Integrals

$\int [f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx$

Example 4: $\int_0^1 (5x^4 + 4x + 7)dx$

Example 5: $\int \frac{x+2\sqrt{x}}{\sqrt{x}}dx$

Example 6:

a. $\int \sqrt[3]{x}(x-4)dx$

b. $\int 2x(x-1)(x+1)dx$

c. $\int \frac{5x^3+x}{x^3}dx$

Other times we are given the derivative and an initial value and we are asked to find the **original function**.

Example 7: Given $f'(x) = 2x + 2,\ f(1) = 5$, find $f(x)$.

Example 8: Given $f''(x) = 6x + 2,\ f'(0) = 2,\ f(0) = 10$, find $f(x)$.

Integrals of Basic Trigonometric Functions:

$\int \sin x\, dx = -\cos x + \mathrm{C}$

$\int \cos x\, dx = \sin x + C$

$\int \sec^2 x\, dx = \tan x + C$

$\int \csc^2 x\, dx = -\cot x + C$

$\int \sec x \tan x\, dx = \sec x + C$

$\int \csc x \cot x\, dx = -\csc x + C$

Example 9: $\int \sin x\, (\csc x + \cot x)\, dx$

Example 10: $\int_0^{\pi/3} \sec x \tan x\, dx$

Example 11: Given $f'(x) = -3\sin x$, $f(\pi) = -1$, find $f(x)$.

Integral of $\frac{1}{x}$

$$\int \frac{1}{x} dx = \ln|x| + C$$

Example 12: $\int_1^2 \frac{x^2-2}{x} dx$

Integrals of Exponential Functions

$$\int e^x dx = e^x + C$$

$\int a^x dx = \frac{a^x}{\ln a} + C$, where $a > 0,\ a \neq 1$

Example 13: $\int (2^x + 5e^x) dx$

Integrals of the Hyperbolic Functions

$\int \sinh x \, dx = \cosh x + C$

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Recall: $cosh\, x = \frac{e^x + e^{-x}}{2}$; $sinh\, x = \frac{e^x - e^{-x}}{2}$

Example 14: $\int_0^1 2 \sinh x \, dx$

Integrals Resulting in Inverse Trigonometric Functions

$\int \frac{1}{\sqrt{1-x^2}} dx = \arcsin x + C$

$\int \frac{1}{1+x^2} dx = \arctan x + C$

$\int \frac{1}{|x|\sqrt{x^2-1}} dx = \operatorname{arcsec} x + C$

Example 15: $\int_{1/2}^{\sqrt{2}/2} \frac{2}{\sqrt{1-x^2}} dx$

Integrating Piece-wise Defined Functions

Example 16: Let $f(x) = \begin{cases} x+2, & -2 \le x \le 0 \\ 2, & 0 < x \le 1 \\ 4-2x, & 1 < x \le 2 \end{cases}$

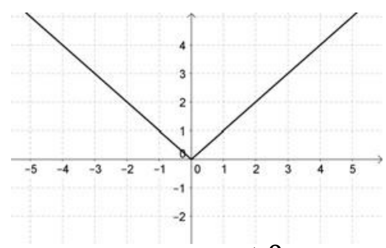
Note how the function changes over the specified domain!

Set-up the integral needed to integrate $\int_{-2}^{2} f(x)dx$.

Integrals Involving Absolute Value

Example 17: $\int_{-1}^{2} |x| dx$

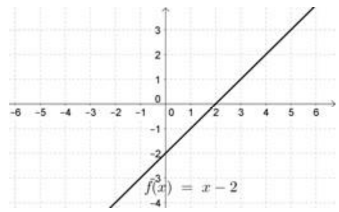
Recall that $y = |x|$ is a piecewise function!



$f(x) = \begin{cases} -x, & x < 0 \\ x, & x \ge 0 \end{cases}$

Example 18:

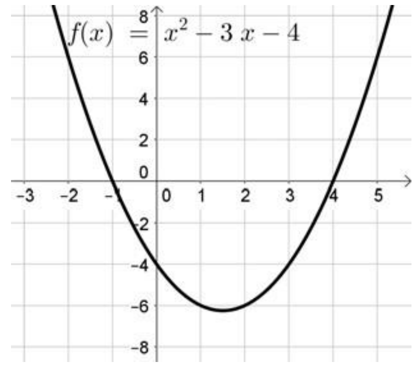
a. $\int_1^4 |x-2|dx$



b. $\int_5^6 |x-2|dx$

Example 19: Set up the following integrals.

a. $\int_1^5 |x^2 - 3x - 4|\, dx$



b. $\int_1^5 |x^2 + 4|\, dx$