## Section 1.3
## Place Value Systems of Numeration

Today the most common type of numeration system is the place value system. A place value system consists of a **base** (a natural number greater than one) and **digits** – a set of symbols representing the numbers from zero to one less than the base.

In this system, the value of the symbol depends on its position in the representation of the number. For example, the 2 in 23 represents 2 tens.

The Hindus in India are credited with the invention of zero and the other symbols used in our system. The Arabs, who traded regularly with the Hindus, also adopted the system, thus the name Hindu-Arabic.

The Hindu-Arabic numerals and the place value system of numeration revolutionized mathematics by making addition, subtraction, multiplication, and division much easier to learn and very practical to use.

In the Hindu-Arabi system, the symbols 0-9 are called **digits** and the base is 10. The base 10 system was developed from counting on fingers, and the word digit comes from the Latin word for fingers.

The positional values in the Hindu-Arabic system are

$$\ldots, 10^5, 10^4, 10^3, 10^2, 10, 1$$

Using the place value rule, we can write a number in expanded form.

$$434 = (4\times10^2)+(3\times10)+(4\times1)$$

The Hindu-Arabic system is used in most of the world today; however, the idea of a place value system goes back as far as the Babylonian system from 2500 BCE. The Mayan civilization in Central America also developed a place value system of numeration in the first millennium CE. This text will examine both of these systems and compare them with the Hindu-Arabic System we use today

**Babylonian Numerals**

The Babylonian’s place value system had a base of 60. It is not a true place value system because it lacked a symbol for zero. The lack of a symbol for zero led to a great deal of ambiguity and confusion.

We’ll use only two symbols to represent these types of numbers, and we’ll write them horizontally, the symbol “ | ” for one and “< ” for ten. These were used to notate the 59 non-zero digits.

The place values in the Babylonian system are

$$\ldots, 60^5, 60^4, 60^3, 60^2, 60, 1$$

In a Babylonian numeral, a gap is left between the characters to distinguish between the various place values. From right to left, the sum of the first group of numerals is multiplied by 1. The sum of the second group is multiplied by 60. The sum of the third group is multiplied by $60^2$, and so on.

Example 1: Write <<     << |||| as a Hindu-Arabic numeral.

Example 2: Write ||     <<< ||     <<<< ||||| as a Hindu-Arabic numeral.

**Converting a Hindu-Arabic numeral to a Babylonian numeral**

1. Find the largest number of the form: …, $60^5$, $60^4$, $60^3$, $60^2$, 60, 1
that is less than the Hindu-Arabic numeral. Then divide this number into the given Hindu-Arabic numeral.
2. Divide the remainder by the next smaller power of 60.
3. Continue in the same pattern by dividing the next remainder by the next smaller power of 60.
4. Once the division by 60 has been performed, the division is done. Arrange the quotients in order in Babylonian symbols followed by the last remainder to obtain the final answer.

Example 3: Write 2519 as a Babylonian numeral.

Example 4: Write 6270 as a Babylonian numeral.

The base 60 system of the Babylonians is called a sexagesimal system. We are not sure why they developed a base 60 system, and a base of 60 seems very unnatural to most modern peoples using base 10. However, the sexagesimal system has had a large influence on modern mathematics.

How many seconds are in a minute?
How many minutes are in an hour?
How many degrees in a circle?

One reason that the base 60 is convenient is that 60 divides evenly by 1,2,3,4,5,6,10,12,15,20 and 30. This makes breaking one unit of 60 into parts or fractions work out nicely, for example, one-third of 60 is 20!

**Mayan Numerals**

Another place value system is the Mayan numeration system. The numbers in this system are written vertically rather than horizontally, with the units position on the bottom.

The number in the bottom row is to be multiplied by 1. The number in the second row from the bottom is to be multiplied by 20. The number in the third row is to be multiplied by $18 \times 20$, or 360. The number in the fourth row is to be multiplied by $18 \times 20^2$, and so on.

It is believed that the Mayans used $18 \times 20$ so that their numeration system would conform to their calendar of 18 months, 20 days each plus 5 "ghost days" to complete the 365 day year.

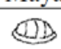
**Place values in the Mayan system**

$$\ldots 18 \times 20^3,\ 18 \times 20^2,\ 18 \times 20,\ 20,\ 1$$

$$144{,}000, \quad 7200, \quad 360, \quad 20,\ 1$$

The digits 0 – 19 of the Mayan systems are formed by the simple groupings of dots and lines. Unlike the Babylonian system, the Mayan system did have a figure for zero.

**Mayan Numerals**

| Number | Mayan | Number | Mayan | Number | Mayan | Number | Mayan |
|---|---|---|---|---|---|---|---|
| 0 |  | 5 | — | 10 | —<br>— | 15 | —<br>—<br>— |
| 1 | • | 6 | •<br>— | 11 | •<br>—<br>— | 16 | •<br>—<br>—<br>— |
| 2 | •• | 7 | ••<br>— | 12 | ••<br>—<br>— | 17 | ••<br>—<br>—<br>— |
| 3 | ••• | 8 | •••<br>— | 13 | •••<br>—<br>— | 18 | •••<br>—<br>—<br>— |
| 4 | •••• | 9 | ••••<br>— | 14 | ••••<br>—<br>— | 19 | ••••<br>—<br>—<br>— |

Example 5: Write the given Mayan numeral as a Hindu-Arabic numeral.

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Example 6: Write the given Mayan numeral as a Hindu-Arabic numeral.

To convert from Hindu-Arabic to Mayan, a similar procedure when converting from Hindu-Arabic to Babylonian is used, except remember the place values are:

$$\ldots 18\times 20^3,\ 18\times 20^2,\ 18\times 20,\ 20,\ 1$$

$$144{,}000, \quad 7200, \quad 360, \quad 20,\ 1$$

Example 7: Write 4020 as a Mayan numeral.

Example 8: Write 6133 as a Mayan numeral.