Section 2.1
Counting Techniques

Combinatorics is the study of the number of ways a set of objects can be arranged, combined, or chosen; or the number of ways a succession of events can occur. Each result is called an **outcome**. The collection of all possible outcomes is the **sample space**. An **event** is a subset of outcomes. When several events occur together, we have a **compound event**.

The **Fundamental Counting Principle** states that the total number of a ways a compound event may occur is $n_1 \cdot n_2 \cdot n_3 \cdot \ldots \cdot n_i$ where n_1 represents the number of ways the first event may occur, n_2 represents the number of ways the second event may occur, and so on.

Example 1: The Burger Bar offers the following items on its menu:

Burger	Sides	Beverages	Desserts
Single Meat	Fries	Tea	Cheesecake
Double Meat	Onion Rings	Coffee	Brownie
	Fruit Bowl	Soda	Cookie
	Cheddar Peppers		Ice Cream Cone

If a customer chooses 1 item from each category, how many meals can be made? List 1 meal possible.

Example 2: A license plate consists of 3 letters followed by 4 digits. How many license plates are possible if the first letter cannot be O, repetition of letters is allowed, but digits may not repeat?

Example 3: How many ways can the letters of the word VOWEL be arranged if the first letter cannot be a vowel?

Permutations

n-Factorial: For any natural number n, $n! = n(n-1)(n-2)\cdots 3\cdot 2\cdot 1$.
0! = 1
R command: factorial( )

A **permutation** of a set is arranging the elements of the set with regard to order.
Example: My previous pin number was 2468, now it's 8642.

Formula: ${}_nP_r = P(n,r) = \frac{n!}{(n-r)!}$, $r \leq n$, where n is the number of distinct objects and r is the number of distinct objects taken r at a time.

Example 4: Seven people arrive at a ticket counter at the same time to buy concert tickets. In how many ways can they line up to purchase their tickets?

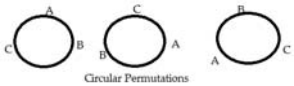
Command: Answer:

Example 5: In how many ways can 3 of the six symbols, @, &, %, $, *, # be arranged on an ID tag?

Command: Answer:

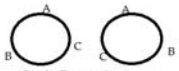
Circular Permutations

Let's say we have the following situation…



Circular Permutations

How can persons *A, B, C* be arranged around a circle?
Not in the three ways as shown above because each one of *A, B, C* has the same neighbor!
Without changing neighbor**, only changing seats will not change the circular permutation**.
Change neighbors and you will change the circular permutation. As follows:



Circular Permutations

So, three persons *A, B, C* can only be arranged in 2 ways around a circle. Hence, *n* different things can be arranged **around a circle in (*n* – 1)! ways**. Whereas, *n* different things can be arranged **in a line n! ways**.
http://www.math-for-all-grades.com/CircularPermutation.html

Example 6: In how many ways can 12 people be seated around a circular table?
Command: Answer:

Formula: Permutations of n objects, not all distinct

Given a set of n objects in which n_1 objects are alike and of one kind, n_2 objects are alike and of another kind,…, and, finally, n_r objects are alike and of yet another kind so that

$$n_1 + n_2 + ... + n_r = n$$

then the number of permutations of these *n* objects taken *n* at a time is given by

$$\frac{n!}{n_1!n_2!\cdots n_r!}$$

Example 7: How many arrangements can be made using all of the letters in the word MISSISSIPPI?
n = total number of objects =
letter *occurs*
M =>
I =>
S =>
P =>
Command: Answer:

Combinations

A **combination** of a set is arranging the elements of the set without regard to order.

Example: The marinade for my steak contains soy sauce, worchester sauce and a secret seasoning.

Formula: ${}_nC_r = C(n,r) = \frac{n!}{r!(n-r)!}$, $r \leq n$, where n is the number of distinct objects and r is the number of distinct objects taken r at a time. R Command: choose(n, r)

Example 8: An organization needs to make up a social committee. If the organization has 25 members, in how many ways can a 10 person committee be made?

Command: Answer:

Example 9: A committee of 16 people, 7 women and 9 men, is forming a 7- member subcommittee that must consist of 3 women and 4 men. In how many ways can the subcommittee be formed?

Command: Answer:

Example 10: A tumbler contains 30 cubes, in which 10 are blue, 10 are yellow, and 10 are red. The balls of each color are lettered A – J. You choose 6 balls at random from the tumbler. How many selections consist of exactly 3 balls with the same letter?

Command: Answer:

Try this one: Five cards are drawn from a well-shuffled 52 card deck.

a. In how many ways can the five cards be drawn?

Command: Answer:

b. In how many ways can four Queens be drawn?

Command: Answer:

c. In how many ways can four Queens or four Kings be drawn?

Command: Answer:

d. In how many ways can any four of a kind be drawn?

Command: Answer: