Section 2.2
Sets and Venn Diagrams

A **set** is a collection of objects. Two sets are equal if they contain the same elements.

The **Universal set** is the set of interest in a particular discussion.

Set A is a **subset** of set B if every element that is in set A is also in set B. The notation for this is $A \subseteq B$.

Set A is a **proper subset** of set B if every element that is in set A is also in set B *and* there is at least one element in set B that is not in set A. The notation for this is $A \subset B$.

The **union** of A and B, which is written as $A \cup B$, is the set of all elements that belong either to set A or to set B or to both A and B. *Key words in word problems will be: "or" or "either and or both"*

The **intersection** of A and B, which is written as $A \cap B$, is the set of all elements that belong to both set A and set B. *Key words in word problems will be: "and", "both", "but","nor"*

If the intersection of two sets is empty (the empty set is denoted by $\varnothing$), then the sets are **disjoint** or **mutually exclusive** and we write $A \cap B = \varnothing$.

Examples of disjoint sets: Choosing a King and a Queen from a deck of cards, Rolling a pair of six-sided dice and getting an even and an odd number

The **complement of set A**, which is written as A^c, is the set of all elements that are in the universal set but are not in set A. *Key word in word problems will be: "not"*

Some Useful Properties

$U^c = \varnothing$ $\qquad$ $\varnothing^c = U$ $\qquad$ $(A^c)^c = A$

$(A \cup B)^c = A^c \cap B^c$ $\qquad$ $(A \cap B)^c = A^c \cup B^c$

Example 1: Let $A = \{2,4,6\}$, $B = \{3,6,9\}$, $C = \{0,6,7\}$, and $U = A \cup B \cup C$

a. Find A^c.

$U = \{0,2,3,4,6,7,9\}$

$= \{0,3,7,9\}$

b. Find $A \cap C$.

$= \{6\}$

$A = \{2,4,6\}$ $B = \{3,6,9\}$ $C = \{0,6,7\}$

$U = \{0,2,3,4,6,7,9\}$

c. Find $(A \cup C)^c$

$A \cup C = \{0,2,4,6,7\}$

$(A \cup C)^c = \{3,9\}$

d. Is $C \subset U$?

YES!

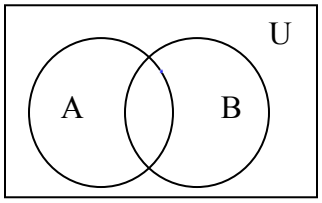
e. Find $C \cap (A \cup B^c)^c$ $= C \cap (A^c \cap B)$

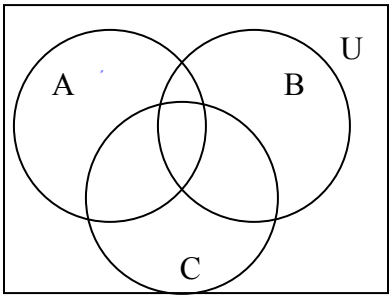
$A^c = \{0,3,7,9\}$

$A^c \cap B = \{3,9\}$ $C \cap (A^c \cap B) = \emptyset$

A **Venn diagram** is a visual representation of sets.

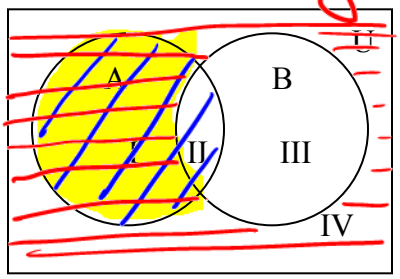
Some look like:





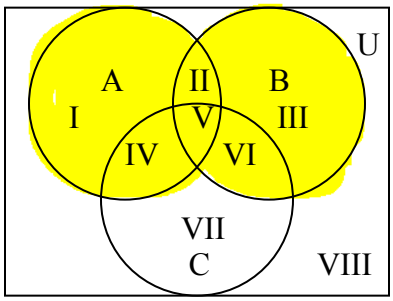
Example 2: Use shading to state the region(s) that represent(s) the given set. (Assume the given sets are not disjoint. This is obvious from the Venn diagrams.)

a. $(A \cap B^c)$ ← strictly A!



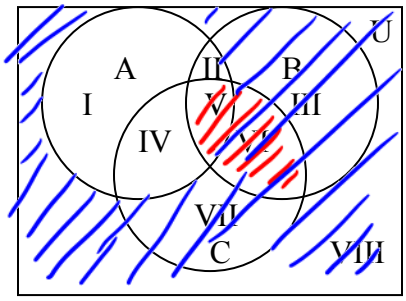
I

b. $(A \cup B)^c$



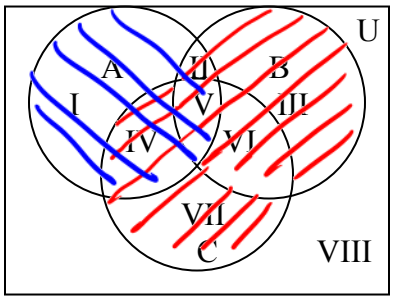
VII + VIII

c. $(A^c \cup (B \cap C))$



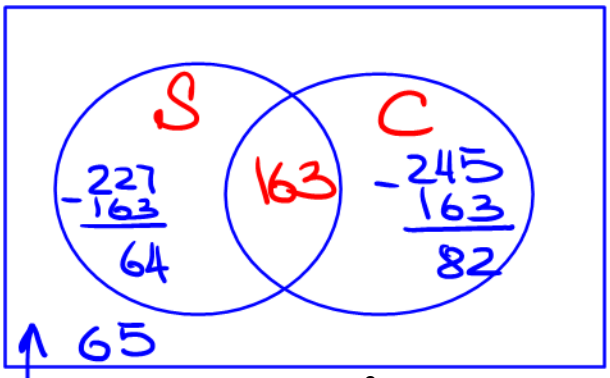
All except I, II, and IV

d. $(A \cap (B \cup C))$



II, IV, and V

Example 3: In a survey of 374 coffee drinkers it was found that 227 take sugar, 245 take cream, and 163 take sugar and cream with their coffee. Use a Venn diagram to answer the following questions.



a. How many take sugar but not cream with their coffee?

64

b. How many take sugar or cream?

64 + 163 + 82 = 309

374 − 64 − 163 − 82 = 65