

Section 2.4 General Probability Rules

Two events are **independent** if knowing that one occurs does not change the probability that the other occurs. Note: This is not the same as sets that are disjoint or mutually exclusive.

If E and F are independent events, then $P(E \cap F) = P(E)P(F)$

Example 1: Determine if events A and B are independent. $P(A) = 0.02$, $P(B) = 0.5$,
 $P(A \cap B) = 0.01$

Check if $P(A \cap B) = P(A)P(B)$

$$0.01 \stackrel{?}{=} 0.02(.5) \checkmark \quad \text{YES} \quad \text{A \& B are independent.}$$

Dependent events, the occurrence of one event **does** have an effect on the occurrence of the other event. The probability $P(E | F)$ is read "the probability of event E given event F had already occurred".

If E and F are independent, then $P(E | F) = P(E)$.

If events E and F are **dependent** then $P(E | F) = \frac{P(E \cap F)}{P(F)}$, and if we cross multiply we get that

$$P(E \cap F) = P(F) \cdot P(E|F)$$

Example 2: Given $P(A) = 0.26$, $P(B) = 0.58$, and $P(A \cap B) = 0.02$. Find $P(B|A)$.

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{.02}{.26} = .0769$$

Example 3: A group of 100 randomly chosen students were asked whether they like Hawaiian Punch or Tang. The probability that a randomly selected student likes Hawaiian Punch (the event H) is $P(H) = 0.75$ and the probability that a randomly selected student likes Tang (the event T) is $P(T) = 0.45$. The probability that a randomly selected student likes **both** kinds of drinks is $0.35 = P(H \cap T)$

a. Find the probability that a randomly selected student likes Tang, **given** that he/she likes Hawaiian Punch.

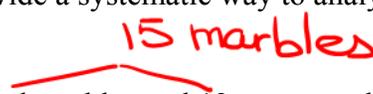
$$P(T|H) = \frac{P(H \cap T)}{P(H)} = \frac{.35}{.75} = .4667$$

b. Find the probability that the selected student likes Tang **or** Hawaiian Punch.

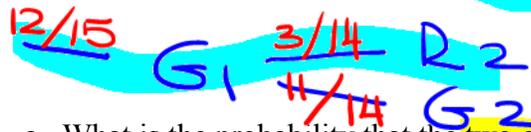
$$P(T \cup H) = P(T) + P(H) - P(T \cap H) \\ = .45 + .75 - .35 = .85$$

$$P(H \cup T) = P(T \cup H) \quad P(H \cap T) = P(T \cap H)$$

When experiments consist of two or more trials and things are chosen one at a time, with or without replacement, **tree diagrams** provide a systematic way to analyze different types of probabilities.



Example 4: Suppose a box contains 3 red marbles and 12 green marbles. Two marbles are chosen at random from the box in succession and without replacement.



and ← multiply!
R1R2

a. What is the probability that the **two chosen are red?**

$$P(R1R2) = \frac{3}{15} \cdot \frac{2}{14} = .0286$$

b. What is the probability that **one is green and the other is red?**

add!
R1G2 or G1R2

$$\frac{3}{15} \cdot \frac{12}{14} + \frac{12}{15} \cdot \frac{3}{14} = .3429$$

Example 5: 1000 students were asked to give their favorite subject and favorite movie. The results are recorded in this two-way table:

	Math	Science	English	Social Studies	Marginal Distributions
Avatar	66	70	40	35	211
Godzilla	54	75	60	30	219
King Kong	35	50	80	90	255
Blade	45	40	60	100	245
Star Trek	10	5	20	35	70
Marginal Distributions	210	240	260	290	1000

a. What is the probability that someone likes **King Kong**?

$$\frac{255}{1000} = .255$$

b. What is the probability that someone likes **both Science and Godzilla**?

$$\frac{75}{1000} = .075$$

c. What percent of people who like Blade also like English?

$$\frac{60}{245} = .245 \text{ or } 24.5\%$$