Section 2.4
General Probability Rules

Two events are **independent** if knowing that one occurs does not change the probability that the other occurs. Note: This is not the same as sets that are disjoint or mutually exclusive.

If E and F are independent events, then $P(E \cap F) = P(E)P(F)$

Example 1: Determine if events A and B are independent. $P(A) = 0.02$, $P(B) = 0.5$, $P(A \cap B) = 0.01$

Check if $P(A \cap B) = P(A)P(B)$

$0.01 \stackrel{?}{=} 0.02(.5)$ ✓ YES

A&B are independent.

Dependent events, the occurrence of one event does have an effect on the occurrence of the other event. The probability $P(E \mid F)$ is read "the probability of event E given event F had already occurred".

If E and F are independent, then $P(E \mid F) = P(E)$.

If events E and F are dependent then $P(E \mid F) = \dfrac{P(E \cap F)}{P(F)}$, and if we cross multiply we get that

$P(E \cap F) = P(F) \cdot P(E]F)$.

Example 2: Given $P(A) = 0.26$, $P(B) = 0.58$, and $P(A \cap B) = 0.02$. Find $P(B|A)$.

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)} = \frac{.02}{.26} = \boxed{.0769}$$

Example 3: A group of 100 randomly chosen students were asked whether they like Hawaiian Punch or Tang. The probability that a randomly selected student likes Hawaiian Punch (the event H) is $P(H) = 0.75$ and the probability that a randomly selected student likes Tang (the event T) is $P(T) = 0.45$. The probability that a randomly selected student likes both kinds of drinks is $0.35. = P(H \cap T)$

a. Find the probability that a randomly selected student likes Tang, given that he/she likes Hawaiian Punch.

$$P(T|H) = \frac{P(H \cap T)}{P(H)} = \frac{.35}{.75} = \boxed{.4667}$$

b. Find the probability that the selected student likes Tang or ($\cup$) Hawaiian Punch.

$$P(T \cup H) = P(T) + P(H) - P(T \cap H)$$
$$= .45 + .75 - .35 = \boxed{.85}$$

$P(H \cup T) = P(T \cup H) \qquad P(H \cap T) = P(T \cap H)$

When experiments consist of two or more trials and things are chosen one at a time, with or without replacement, tree diagrams provide a systematic way to analyze different types of probabilities.

15 marbles

Example 4: Suppose a box contains 3 red marbles and 12 green marbles. Two marbles are chosen at random from the box in succession and without replacement.

3/15 R_1 — 2/14 R_2; 12/14 G_2

12/15 G_1 — 3/14 R_2; 11/14 G_2

a. What is the probability that the two chosen are red? $R_1 \cap R_2$ and ← multiply!

$$P(R_1 \cap R_2) = \frac{3}{15} \cdot \frac{2}{14} = \boxed{.0286}$$

b. What is the probability that one is green and the other is red? R_1G_2 or G_1R_2 add!

$$\frac{3}{15} \cdot \frac{12}{14} + \frac{12}{15} \cdot \frac{3}{14} = \boxed{.3429}$$

Example 5: 1000 students were asked to give their favorite subject and favorite movie. The results are recorded in this two-way table:

	Math	**Science**	**English**	**Social Studies**	**Marginal Distributions**
Avatar	66	70	40	35	211
Godzilla	54	75	60	30	219
King Kong	35	50	80	90	255
Blade	45	40	60	100	245
Star Trek	10	5	20	35	70
Marginal Distributions	210	240	260	290	**1000**

a. What is the probability that someone likes King Kong?

$255/1000 = \boxed{.255}$

b. What is the probability that someone likes both Science and Godzilla? $\cap$

$75/1000 = \boxed{.075}$

c. What percent of people who like Blade also like English?

$60/245 = .245$ or $\boxed{24.5\%}$