Section 3.1
Random Variables

A **random variable** is a variable whose value is a numerical outcome of a random experiment. It assigns one and only one numerical value to each point in the sample space for an experiment.

A **discrete random variable** is one that can assume a countable number of possible values. *For example, the number of people in an experiment.*

A **continuous random variable** can assume any value in an interval on the number line. *For example, the life-time of a light bulb.*

A **probability distribution table of *X*** consists of all possible values of a discrete random variable with their corresponding probabilities.

Notation: Capital letters of the alphabet are used to denote random variables. P(X = *x*) is read as, "The probability the random variable X assumes the value *x*."

Recall that the **mean**,or **expected value**, is a measure of the center of the distribution. The mean of a random variable *X* is found with the following formula:

$$\mu_X = E(X) = x_1 p_1 + x_2 p_2 + \ldots + x_n p_n$$

Example 1: A furniture store is having a sale on sofas and you're going to buy one. The advertisers know that buyers get to the store and that 1 out of 4 buyers change to a more expensive sofa than the one in the sale advertisement. Let X be the cost of the sofa. What is the average cost of a sofa if the advertised sofa is \$200 and the more expensive sofa is \$375?

Price	P
200	3/4
375	1/4

$$E(X) = 200\left(\tfrac{3}{4}\right) + 375\left(\tfrac{1}{4}\right) = \boxed{\$243.75}$$

Example 2: Suppose you want to play a carnival game that costs 5 dollars each time you play. If you win, you get \$100. The probability of winning is 3 out of 100. What is the expected value of the amount that you, the player, stand to gain?

Gain	P
95	3/100
-5	97/100

$$E(X) = 95\left(\tfrac{3}{100}\right) + (-5)\left(\tfrac{97}{100}\right)$$

$$= -2$$

On average, you will loose \$2

Also recall that the **variance** and **standard deviation** are measures of the spread of a distribution. The formula for the variance of a random variable X can be found using the following:

sigma $\rightarrow$

$$\sigma_X^2 = Var(X) = (x_1 - \mu)^2 p_1 + (x_2 - \mu)^2 p_2 + \ldots + (x_n - \mu)^2 p_n$$
$$= \sum_{i=1}^{n} (x_i - \mu)^2 p_i$$

An alternate formula is $\boxed{\sigma_X^2 = Var(X) = E(X^2) - \left(E(X)\right)^2.}$

The formula for standard deviaiton is

$$\sigma_X = \sqrt{Var(X)}$$

Example 3: Suppose you are given the following distribution table:

X^2 1 4 9 16

X	1	2	3	4
P(X)	**0.31**	**0.22**	**0.15**	x_4

$x_4 = 1 - (.31 + .22 + .15)$
$= .32$

$\uparrow$ .32

Find the following:

a. P(2 < X ≤ 4) $= P(3 \le X \le 4)$

$= P(X = 3) + P(X = 4)$

$= .15 + .32 = \boxed{.47}$

b. Find the smallest x such that P(X ≥ x) < 0.5?

$P(X \ge 1) = 1$ $\quad 1 \not< .5$

$P(X \ge 2) = .22 + .15 + .32 = .69$ $\quad .65 \not< .5$

$P(X \ge 3) = .15 + .32 = .47$ $\quad .47 < .5$ ✓

$\boxed{x = 3}$

c. Find the standard deviation.

$Var(X) = E(X^2) - (E(X))^2$

$E(X) = 1(.31) + 2(.22) + 3(.15) + 4(.32) = 2.48$

$E(X^2) = 1(.31) + 4(.22) + 9(.15) + 16(.32) = 7.66$

$Var = 7.66 - (2.48)^2 = 1.5096$

$\sigma = \sqrt{Var(X)} = \sqrt{1.5096} = \boxed{1.2287}$

16

7G 9B

Example 4: You have a group of students and each must do a presentation. You have 7 girls and 9 boys. You choose two students at random to do their presentation.

a. Create a probability distribution for the number of girls you chose.

x	0	1	2
P(X = x)	$\frac{C(7,0)\,C(9,2)}{C(16,2)}$	$\frac{C(7,1)\,C(9,1)}{C(16,2)}$	$\frac{C(7,2)\,C(9,0)}{C(16,2)}$
	.3	.525	.175

b. Find the mean for your distribution.

$$\mu = 0(.3) + 1(.525) + 2(.175) = .875$$

$\approx \boxed{1 \text{ girl}}$

Rules for Means and Variances

Suppose X is a random variable and we define W as a new random variable such that $W = aX + b$, where a and b are real numbers. We can find the mean and variance of W with the following formula:

$$E(Y) = E(aX + b) = aE(X) + b$$

$$\sigma_Y^2 = Var(Y) = Var(aX + b) = a^2 Var(X)$$

Likewise, we have a formula for random variables that are combinations of two or more other independent random variables. Let X and Y be independent random variables,

$$E(X + Y) = E(X) + E(Y)$$

$$\sigma_{X+Y}^2 = Var(X + Y) = Var(X) + Var(Y)$$

and

$$E(X - Y) = E(X) - E(Y)$$

$$\sigma_{X-Y}^2 = Var(X - Y) = Var(X) + Var(Y) \quad \Leftarrow \text{YES, IT'S PLUS!}$$

Var = 9!

Example 5: Suppose you have a distribution, X, with μ =22 and σ = 3. Define a new random variable $Y = 4X + 1$.

a. Find the mean of Y.

$$E(Y) = E(aX + b) = aE(X) + b$$

$= 4(22) + 1$

$= \boxed{89}$

b. Find the standard deviation of Y.

$$\sigma_Y = \sqrt{a^2 Var(X)}$$

$= \sqrt{4^2 \cdot 9}$

$= \boxed{12}$