Section 3.2
Binomial Distributions

A **Bernoulli Trial** is a random experiment with the following features:
1. The outcome can be classified as either a success or a failure (only two options and each is mutually exclusive).
2. The probability of success is p and probability of failure is $1 - p$.

A **Bernoulli random variable** is a variable assigned to represent the successes in a Bernoulli trial.

If we wish to keep track of the number of successes that occur in repeated Bernoulli trials, we use a **binomial random variable**. Assuming there are n trials, then the random variable takes on the numbers $\{0, 1, 2, \ldots, n\}$.

A **binomial experiment** occurs when the following conditions are met:
1. Each trial can result in one of only two mutually exclusive outcomes (success or failure).
2. There are a fixed number of trials.
3. Outcomes of different trials are independent.
4. The probability that a trial results in success is the same for all trials.

Binomial probabilities are calculated with the following formula:

$$P(X = k) = C(n,k) \cdot p^k \cdot (1 - p)^{n-k}$$

where X = binomial random variable, n = **whole** number of trials, k = number of successes, and p is the probability of success.

R-Studio Commands:

$P(X = k) =$ dbinom(k,n, p)

$P(X \le k)$ = pbinom(k,n,p)

$P(X > k) = 1 - P(X \le k)$ = 1 – pbinom(k,n,p)

Example 1: Let X be a binomial random variable with probability success 0.32 and 10 independent trials. Calculate each of the following using R-Studio

a. P(X = 5)
Command: Answer:

b. P(X $\le$ 2)
Command: Answer:

c. P(X < 2)
Command: Answer:

d. $P(X > 8)$

Command: Answer:

e. $P(X \geq 8)$

Command: Answer:

f. $P(3 \leq X \leq 6)$

Command: Answer:

Example 2: A fair coin is flipped 30 times. Find the probability that the coin comes up tails:
a. exactly 12 times?

Command: Answer:

b. less than 12 times?

Command: Answer:

c. 11 or more times?

Command: Answer:

Binomial Distribution Formulas for Mean, Variance and Standard Deviation

$$\mu = E(X) = np$$

$$\sigma^2 = np(1-p)$$

$$\sigma = \sqrt{np(1-p)}$$

Example 3: Suppose it is known that 80% of the people exposed to the flu virus will contract the flu. Out of a family of five exposed to the virus, what is the probability that:

a. at least two will get the flu?

Command: Answer:

b. between two and four, inclusive, will get the flu?

Command: Answer:

c. Find the mean and standard deviation of this distribution.

$\mu = E(X) = np$

$\sigma = \sqrt{np(1-p)}$

Note: This is a binomial distribution since:
The trials: are fixed, each is independent and the probability of success for each is the same.