Section 3.2 **Binomial Distributions**

A **Bernoulli Trial** is a random experiment with the following features:

1. The outcome can be classified as either a success or a failure (only two options and each is mutually exclusive).

2. The probability of success is p and probability of failure is 1 - p.

A Bernoulli random variable is a variable assigned to represent the successes in a Bernoulli trial.

If we wish to keep track of the number of successes that occur in repeated Bernoulli trials, we use a **binomial random variable**. Assuming there are *n* trials, then the random variable takes on the numbers $\{0, 1, 2, ..., n\}$.

A **binomial experiment** occurs when the following conditions are met:

- 1. Each trial can result in one of only two mutually exclusive outcomes (success or failure).
- 2. There are a fixed number of trials.
- 3. Outcomes of different trials are independent.
- 4. The probability that a trial results in success is the same for all trials.

Binomial probabilities are calculated with the following formula:

$$P(X = k) = C(n,k) \cdot p^k \cdot (1-p)^{n-k}$$

where X = binomial random variable, n = whole number of trials, k = number of successes, and p is the probability of success.

| R-Studio Commands: | P(x <k)< th=""><th></th><th></th></k)<> | | |
|--|---|---|--|
| = P(X = k) = dbinom(k, n, p) | P(x=k) | | |
| $\implies P(X \leq k) = \underline{pbinom(k,n,p)}$ | P(X ≥ K) | | |
| $P(X > k) = 1 - P(X \le k) = 1$ | $- \underline{pbinom}(k,n,p)$ | h | |

Example 1: Let X be a binomial random variable with probability success 0.32 and 10 independent trials. Calculate each of the following using R-Studio a. P(X = 5)

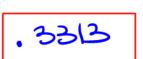
Command:
$$dbinom(5, 10, .32)$$

b. $P(X \le 2)$

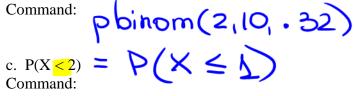
Answer:

Answer:

Answer:



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Command:

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d. $P(X > 8) = 1 - P(X \le 8)$ Command: 1 - pbinom(8, 10, .32)Answer: e. $P(X \ge 8) = \int -P(X \le 7)$ Command: Answer ,0025 1-pbinom (7,10,.32) f. $P(3 \le X \le 6)$ P(X≤6) - P (X≤2) Command: Answer: binom(6,10,.32) - pbinom(2,10,32).653 Example 2: A fair coin is flipped 30 times. Find the probability that the coin comes up tails: a. exactly 12 times? SICCESS P(X = 1) $\frac{1}{2} = \frac{1}{2} + \frac{1}$ Command: Answer: . 0806 b. less than 12 times? $P(X < I_2) = P(X \le I_1)$ Command: Answer: 1002 c. 11 or more times? P(X≥11)=1-P(x<11)=1-P(x≤10) Command: Answer: 1-pbinom (10,30,.5) 9506

Binomial Distribution Formulas for Mean, Variance and Standard Deviation

| $\mu = E(X) = np$ |
|---------------------------|
| $\sigma^2 = np(1-p)$ |
| $\sigma = \sqrt{np(1-p)}$ |

Example 3: Suppose it is known that 80% of the people exposed to the flu virus will contract the flu. Out of a family of five exposed to the virus, what is the probability that: n = 5 a. at least two will get the flu? 8 = -9P(X≥2) = I-P(X≤1 (k, n, p)Command: Answer: 1- phinom (1, .9932 b. between two and four, inclusive, will get the flu? P(2=X=4) O $-P(X \leq 4) - P(X \leq 1)$ Command: Answer: pbinom(4,5,.8) - pbinom(1,5,.8)6656 c. Find the mean and standard deviation of this distribution. h=5 p=.8 $\mu = E(X) = np = 5(.8) = 4$ $\sigma = \sqrt{np(1-p)}$ 15(.8)(.2) = .8944 S9rt (5*.8*.2) or (5*.8*.2)1 Note: This is a binomial distribution since: The trials: are fixed, each is independent and the probability of success for each is the same.