Section 3.2
Binomial Distributions

A **Bernoulli Trial** is a random experiment with the following features:

1. The outcome can be classified as either a success or a failure (only two options and each is mutually exclusive).
2. The probability of success is p and probability of failure is $1 - p$.

A **Bernoulli random variable** is a variable assigned to represent the successes in a Bernoulli trial.

If we wish to keep track of the number of successes that occur in repeated Bernoulli trials, we use a **binomial random variable**. Assuming there are n trials, then the random variable takes on the numbers $\{0, 1, 2, \ldots, n\}$.

A **binomial experiment** occurs when the following conditions are met:

1. Each trial can result in one of only two mutually exclusive outcomes (success or failure).
2. There are a fixed number of trials.
3. Outcomes of different trials are independent.
4. The probability that a trial results in success is the same for all trials.

Binomial probabilities are calculated with the following formula:

$$\boxed{P(X = k) = C(n,k) \cdot p^k \cdot (1-p)^{n-k}}$$

where X = binomial random variable, n = **whole** number of trials, k = number of successes, and p is the probability of success.

R-Studio Commands:

$P(X < k)$

$P(X \geq k)$

$P(X = k) =$ dbinom(k,n, p)

$P(X \leq k) =$ pbinom(k,n,p)

$P(X > k) = 1 - P(X \leq k) =$ 1 – pbinom(k,n,p)

p n

Example 1: Let X be a binomial random variable with probability success 0.32 and 10 independent trials. Calculate each of the following using R-Studio

a. P(X = 5)
Command: dbinom(5,10,.32)
Answer: $\boxed{.1229}$

b. P(X ≤ 2)
Command: pbinom(2,10,.32)
Answer: $\boxed{.3313}$

c. P(X < 2) = P(X ≤ 1)
Command: pbinom(1,10,.32)
Answer: $\boxed{.1206}$

d. $P(X > 8) = 1 - P(X \le 8)$

Command: 1- pbinom(8,10,.32) Answer: $\boxed{.0003}$

e. $P(X \ge 8) = 1 - P(X \le 7)$

Command: Answer:

1- pbinom(7,10,.32) $\boxed{.0025}$

f. $P(3 \le X \le 6)$

2 3 4 5 6

$P(X \le 6) - P(X \le 2)$

Command: Answer:

pbinom(6,10,.32) – pbinom(2,10,.32) = $\boxed{.6532}$

n = 30 p = 1/2

Example 2: A fair coin is flipped 30 times. Find the probability that the coin comes up tails:

success

a. exactly 12 times?

$P(X = 12)$

Command: (k, n, p) Answer:

dbinom(12, 30, .5) .0806

b. less than 12 times?

$P(X < 12) = P(X \le 11)$

Command: Answer:

pbinom(11,30,.5) .1002

c. 11 or more times?

$P(X \ge 11) = 1 - P(X < 11) = 1 - P(X \le 10)$

Command: Answer:

1 - pbinom(10,30,.5) .9506

Binomial Distribution Formulas for Mean, Variance and Standard Deviation

$$\mu = E(X) = np$$

$$\sigma^2 = np(1-p)$$

$$\sigma = \sqrt{np(1-p)}$$

Example 3: Suppose it is known that 80% of the people exposed to the flu virus will contract the flu. Out of a family of five exposed to the virus, what is the probability that:

$n=5$

$p=.8$

a. at least two will get the flu?

$P(X \geq 2) = 1 - P(X \leq 1)$

Command: (k, n, p) Answer:

1- pbinom(1, 5, .8) .9933

b. between two and four, inclusive, will get the flu?

$P(2 \leq X \leq 4)$

0 1 2 3 4 5

$= P(X \leq 4) - P(X \leq 1)$

Command: Answer:

pbinom(4,5,.8) - pbinom(1,5,.8) .6656

c. Find the mean and standard deviation of this distribution.

$n=5$ $p=.8$

$\mu = E(X) = np = 5(.8) = \boxed{4}$

$\sigma = \sqrt{np(1-p)} = \sqrt{5(.8)(.2)} = .8944$

sqrt(5*.8*.2) or (5*.8*.2)^(1/2)

Note: This is a binomial distribution since:
The trials: are fixed, each is independent and the probability of success for each is the same.