Section 4.1
Density Curves

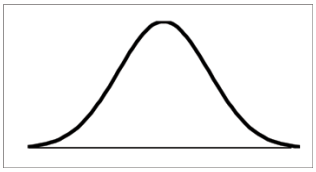
A **density curve** is a graph whose area between it and the *x*-axis is equal to one. These graphs come is a variety of shapes, but the most familiar "normal" graph is bell shaped (see the graphs below). The area under the curve in a range of values indicates the proportion of values in that range (so we'll be able to find probabilities).

So for a continuous random variable (takes on infinitely many values), the probability that *X* is in any given interval is equal to the area between the graph of the function and the *x*-axis over that interval.

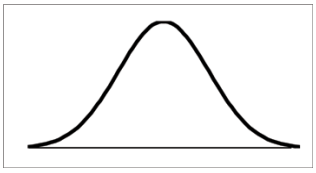
Recall that for a discrete random variable (like those for a binomial and geometric distribution), $P(X < x) \neq P(X \leq x)$. But for continuous random variables, $P(X < x) = P(X \leq x)$. This is due to the fact that the probabilities here deal with area under a curve and above the *x*-axis. Since the area under one single value of *x* has height but no length, $P(X = x) = 0$. Hence, $P(X < x) = P(X \leq x)$.

Let's shade:

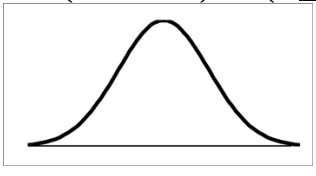
1. $P(X < x) = P(X \leq x)$



2. $P(X > x) = P(X \geq x)$



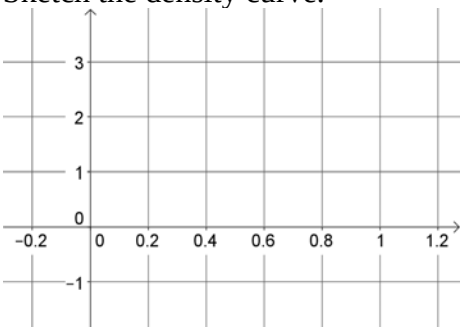
3. $P(a < X < b) = P(a \leq X < b) = P(a < X \leq b) = P(a \leq X \leq b)$



Before we study these types of curves in more detail, let's look at some very simply density curves.

Example 1: Think about a density curve that consists of two line segments. The first goes from the point (0, 1) to the point (0.4, 1). The second goes from (0.4, 1) to (0.8, 2) in the *xy* plane. Let X be the continuous random variable.

Sketch the density curve.



a. What percent of observations fall below 0.4?

b. What is the probability that X lies between 0.4 and 0.8?

c. Find $P(X = 0.4)$.

d. Find $P(X \geq 0.1)$.

Example 2: Consider a uniform density curve (has the same height all the way across) defined for $0 \leq X \leq 10$, where X is the continuous random variable.

Sketch the uniform density curve.

a. What is the probability that X falls below 2?

b. What percent of the observations of X lie between 2 and 5?

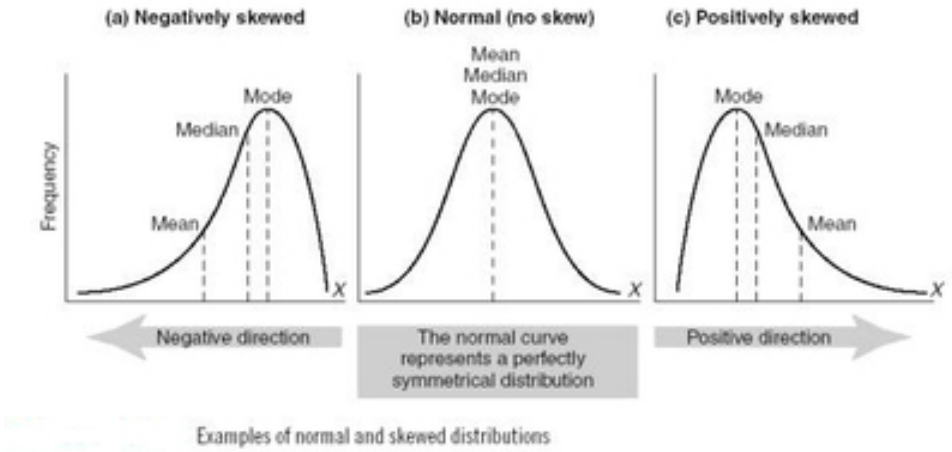
c. Find the median (where the graph splits in half; 50/50).

Example 3: You choose any real number between 0 and 1. What's the probability that your number will be greater than $\frac{17}{19}$? How about $P\left(\frac{1}{8} \le X \le \frac{21}{40}\right)$?

Skewness and Curves

Data can be "skewed", meaning it tends to have a long tail on one side or the other.

In the graphs below: (a) Skewed Left (b) No Skew (c) Skewed Right



Examples of normal and skewed distributions

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If the distribution is symmetric then the mean is equal to the median and the distribution will have zero skewness. A mode of a continuous probability distribution is a value at which the density curve attains its maximum value.

Example 4: Use the given density function to determine which letter represents:
a. Mean b. Median c. Mode