Section 4.3
Standard Normal Calculations

As suggested in the previous section, all normal distributions share many common properties. In fact, if σ is changed to 1 and μ to 0 (center the graph), all normal distributions would be exactly the same. This is called **standardizing**. If x is an observation from a normal distribution with mean μ and standard deviation σ, the **standardized value** of x is called the ***z*-score** and is computed with the formula below.

$$\boxed{\textbf{z-Score:} \quad z = \frac{x-\mu}{\sigma}}$$

A *z*-score tells us how many standard deviations the observed value falls from the mean.

Example 1: The average of a test was 81 with a standard deviation of 3. If a z-score of 1.52 is given, what value for x did this correspond to?

$$3 \cdot 1.52 = \frac{x-81}{\cancel{3}} \cdot \cancel{3}$$

$$4.56 = x - 81$$

$$\boxed{x = 85.56}$$

We can use *z*-scores to "standardize" values that are on different scales to compare them.

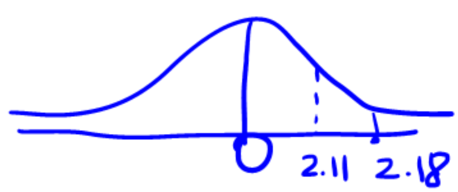
Example 2: Carrie took the ACT and scored 31. Cindy took the SAT and scored 1390. If both tests are normally distributed, who did better? The ACT has a mean of 21.1 and a standard deviation of 4.7. The SAT has a mean of 1010 and a standard deviation of 174.5. Who did better?

Carrie:

$$z = \frac{31-21.1}{4.7} = 2.11$$

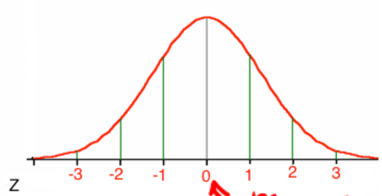
Cindy:

$$z = \frac{1390-1010}{174.5} = 2.18$$



Carrie or Cindy

The **standard normal distribution** is the normal distribution with $N(\mu, \sigma) = N(0,1)$:



mean

If the normal curve is symmetric then which is true?

a. $\boxed{\text{Mean = Median}}$ b. Mean > Median c. Mean < Median

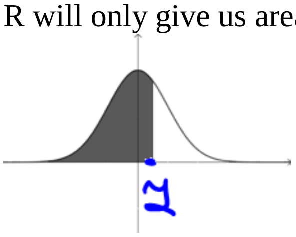
What can be said about the positive z-scores and the negative z-scores?

Positive z-scores lie to the right of the mean.
Negative z-scores lie to the left of the mean.

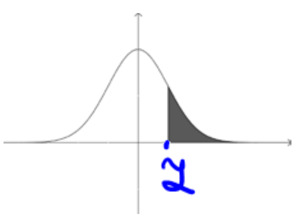
The next thing we'll want to do is to find some probabilities given the standard normal curve. Table A in the appendix of the book can help, but using R is a bit easier. Commands follow…

standard

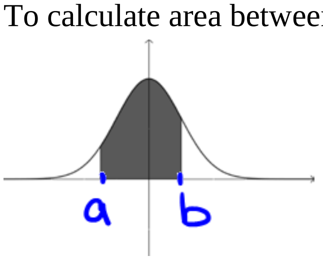
R will only give us area to the left of a value, i.e. $P(Z < z)$. Command: pnorm(z)



To calculate area to the right of a number, $P(Z > z)$, command: 1 – pnorm(z)



To calculate area between two numbers, $P(a < Z < b)$, command: pnorm(b) – pnorm(a)



Example 3: Use R to find the following probabilities.

a. $P(Z < -1.06)$

Command: Answer:

pnorm (-1.06) 0.1446

b. $P(Z > 1.78)$

Command: Answer:

1-pnorm(1.78) .0375

c. $P(-1.03 < Z < 1)$

pnorm(1) - pnorm(-1.03) .6898

Command: Answer:

If we want to use the table for probabilities and are not given z, we must compute the z-score using the formula on page one: $z = \frac{x-\mu}{\sigma}$

normal → μ σ

Example 4: If X has distribution $N(100,15)$, standardize X and use R to find $P(X > 105)$.

st. normal

$P\left(Z > \frac{105-100}{15}\right) = P(Z > .33)$

Command: Answer:

1-pnorm(.33) → .3694

OR

1-pnorm(105,100,15)

Now, let's suppose we know the percentile rank or the probability and want to find the corresponding *z*-score.

We can use Table A and look up the percentile (remember, it shows the area to the left) or we can use the R commands:

- $P(Z < c) = p$, command: qnorm(p)
- $P(Z > c) = p$, command: qnorm($1 - p$)
- $P(-c < Z < c) = p$, command: qnorm($(p+1)/2$)

?

Example 5: Find the value of c so that:

a. $P(Z < c) = 0.7704$

Command: qnorm(.7704)

Answer: .7402

b. $P(Z > c) = 0.006$

Command: qnorm(1-.006)

Answer: 2.5121

c. $P(-c < Z < c) = 0.966$

Command: qnorm $\left(\frac{0.966+1}{2}\right)$

Answer: 2.1201