Section 8.2
Inference for a Population Proportion

For these inferences, p_0 represents the given population proportion.

$H_0 : p = p_0$

$H_a : p \neq p_0$ or $p < p_0$ or $p > p_0$

Conditions:

1. The sample must be an SRS from the population of interest.
2. The population must be at least 10 times the size of the sample.
3. The number of successes must be $n\hat{p} \geq 10$, and the number of failures must be $n(1-\hat{p}) \geq 10$.

Recall, the statistic used for proportions is $\hat{p} = \dfrac{number\ of\ successes}{number\ of\ oberservations}$.

For tests involving proportions that meet the above conditions, we will use the *z* – test statistic where $z = \dfrac{\hat{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}}$.

Example 1: A new shampoo is being test-marketed. A large number of 16-ounce bottles were mailed out at random to potential customers in the hope that the customers will return an enclosed questionnaire. Out of the 1,000 returned questionnaires, 575 indicated that they like the shampoo and will consider buying it when it becomes available on the market. Perform a hypothesis test to determine if the proportion of potential customers is more than 50%.

The conditions check here.

State:

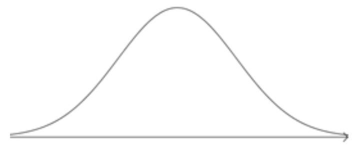
$H_0 : p = p_0 =$

$H_a : p \neq p_0$ or $p < p_0$ or $p > p_0$

$n =$

$\hat{p} = \dfrac{number\ of\ successes}{number\ of\ oberservations} =$

$\alpha =$ Is it one-tailed or two-tailed?



Command: Answer:

Test Statistic:

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} =$$

Fail to reject the null or reject the null?

Let' verify by finding the *p*-value:

Conclusion: We have overwhelming evidence to reject the null hypothesis and conclude that more than 50% are potential customers.

Example 2: Mars Inc. claims that they produce M&Ms with the following distributions:

Brown	30%	Red	20%	Yellow	20%
Orange	10%	Green	10%	Blue	10%

A bag of M&Ms was randomly selected from the grocery store shelf, and the color counts were:

Brown	14	Red	14	Yellow	5
Orange	7	Green	6	Blue	10

Conduct an appropriate test of the manufacturer's claim for the proportion of brown M&Ms.

The conditions check here.

State:

$H_0 : p = p_0 =$

$H_a : p \neq p_0$ or $p < p_0$ or $p > p_0$

$n =$

$$\hat{p} = \frac{number\ of\ successes}{number\ of\ oberservations} =$$

$\alpha =$ Is it one-tailed or two-tailed?

Command: Answer:

Test Statistic:

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1 - p_0)}{n}}} =$$

Fail to reject the null or reject the null?

Let's verify by finding the *p*-value:

Conclusion:

Decision errors can occur when choosing to reject or failing to reject the null hypothesis. There are two types of decision errors; Type I and Type II.

A **Type I error** occurs when you reject the null hypothesis when in fact it is true. This error would correspond to taking action when you would have been better off not doing so. Or when a test is performed and shows an effect, when in fact there is none.

A **Type II error** occurs when you fail to reject the when in fact it is false. The Type II error would correspond to taking no action when you would have been better off taking action. Or when a test is performed and shows no effect, when in fact there is an effect.

The **power** of a test is the probability, α , of rejecting the null, given it is false. The following table can help to identify the types of errors.

	H_0 is true	**H_0 is false**
Reject H_0	Type I error	Decision is correct; Power of a test
Fail to reject H_0	Decision is correct	Type II error

Null Hypothesis	Type I Error / False Positive	Type II Error / False Negative
Person is not guilty of the crime	Person is judged as **guilty** when the person actually **did not** commit the crime (convicting an innocent person)	Person is judged **not guilty** when they actually **did** commit the crime (letting a guilty person go free)

Recall:
A **Type I error** is when a test is performed and shows an effect, when in fact there is none.

A **Type II error** is when a test is performed and shows no effect, when in fact there is an effect.

Example 3: If a doctor told you that you were pregnant, but you were not then this would be a Type I or Type II error?

Example 4: If a doctor tells you that you are not pregnant, when if fact you are pregnant then this would be a Type I or Type II error?