Section 8.3
Comparing Two Means

Two – sample *t* – tests compare the responses to two treatments or characteristics of two populations. There is a separate sample from each treatment or population. These tests are quite different than the matched pairs *t* – test discussed in section 8.1.

How can we tell the difference between dependent and independent populations/samples?

- Dependent: If you were to rearrange one list then it wouldn't match-up nor make sense. *Like Before-After, Pre-test Post-test, etc.*
- Independent: Otherwise.

The null and alternate hypotheses would be:

$H_0 : \mu_1 = \mu_2$

$H_a : \mu_1 \neq \mu_2$ or $\mu_1 > \mu_2$ or $\mu_1 < \mu_2$

Assumptions for a two-sample *t* – test (these are used when the population standard deviations (or variance) are unknown) are:

1. We have two independent SRSs, from two distinct populations and we measure the same variable for both samples.
2. Both populations are normally distributed with unknown means and standard deviations. Or if each given sample size it's at least 30. Or we'll assume normality.

Two-sample *t* – test statistic: $t = \dfrac{(\bar{x}_1 - \bar{x}_2) - \overbrace{(\mu_1 - \mu_2)}^{=0}}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$ $\mu_1 = \mu_2$

The degrees of freedom is equal to the smaller of $n_1 - 1$ and $n_2 - 1$.

Example 1: The president of an all-female school stated in an interview that she was sure that the students at her school studied more, on average, than the students at a neighboring all-male school. The president of the all-male school responded that he thought the mean study time for each student body was undoubtedly about the same and suggested that a study be undertaken to clear up the controversy. Accordingly, independent samples were taken at the two schools with the following results:

School	Sample Size	Mean Study Time (hrs)	Standard deviation (hrs)
All Female	65 $= n_1$	18.56 $= \bar{x}_1$	4.35 $= s_1$
All Male	75 $= n_2$	17.95 $= \bar{x}_2$	4.87 $= s_2$

$\alpha = 0.02$

Determine, at the 2% level of significance, if there is a significant difference between the mean studying times of the students in the two schools based on these samples.

Note: These are independent samples.

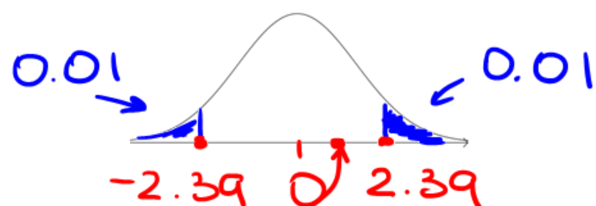
First, make sure to label each *n*, mean and standard deviation above.

State:

$H_0: \mu_1 = \mu_2$

$H_a: \mu_1 \neq \mu_2$

$\alpha =$ 0.02 Is it one-tailed or $\boxed{\text{two-tailed?}}$



These are *t*-tests so don't forget df is equal to the smaller of $n_1 - 1$ and $n_2 - 1$. $df = 65 - 1 = 64$

Command: qt (0.01, 64) Answer: −2.386037

Test Statistic:

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - \cancel{(\mu_1 - \mu_2)}}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{18.56 - 17.95}{\sqrt{\frac{4.35^2}{65} + \frac{4.87^2}{75}}} = 0.7827$$

0

$\boxed{\text{Fail to reject the null}}$ or reject the null?

Find the p-value:
(*This is the probability that we got the data assuming the null is true.*)

$2 * P(t < 0.7827)$

$= 2 * pt(0.7827, 64)$

$= 0.4367 > \alpha = 0.02$

The p-value tells us that 44% chance (probability) that there is a difference in the mean study times for the two schools (assuming the null is true). Since we fail to reject the null then we can say that the observed data can be explained by chance alone.

Conclusion: Based on a 2% significance level (or based on a 98% certainty), we'll fail to reject the null which states no difference in the mean study times for the two schools.

Example 2: A study was conducted to determine whether remediation in basic mathematics enabled students to be more successful in an elementary statistics course. Samples of final exam scores were taken from students who had remediation and from students who did not. Here are the results of the study:

	Remedial	Non-remedial
Sample size	100 $= n_1$	40 $= n_2$
Mean Exam Grade	83.0 $= \bar{x}_1$	76.5 $= \bar{x}_2$
Std Dev for Exam	2.76 $= s_1$	4.11 $= s_2$

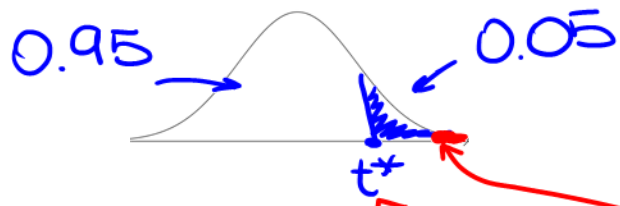
Test, at the 5% level, whether the remediation helped the students to be more successful.

$\alpha = 0.05$

First, make sure to label each n, mean and standard deviation above.

State:
$H_0: \mu_1 = \mu_2$
$H_a: \mu_1 > \mu_2$

$\alpha = 0.05$ Is it $\boxed{\text{one-tailed}}$ or two-tailed?



These are t-tests so don't forget df is equal to the smaller of $n_1 - 1$ and $n_2 - 1$. df = 39

Command: qt(0.95, 39) Answer: 1.68

Test Statistic:

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - \cancel{(\mu_1 - \mu_2)}^{\,0}}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{83 - 76.5}{\sqrt{\frac{2.76^2}{100} + \frac{4.11^2}{40}}} = 9.206$$

Fail to reject the null or $\boxed{\text{reject the null?}}$

Find the p-value:

$$P(t > 9.206) = 1 - pt(9.206, 39)$$
$$= 1.2622 \times 10^{-11} < \alpha = 0.05$$

The p-value is the chance (probability) that remediation helped the students be more successful (assuming the null is true). Since we reject the null in favor of the alternate then we can say that there is something besides chance alone that gave us the observed data.

95% certainty

Conclusion: Based on a 5% significance level, we'll reject the null which states that there is no difference in favor of the alternate which states that remediation helped the students be more successful.