Section 8.4
Comparing Two Proportions

When comparing two population proportions in an inference test, we use a **two-sample *z* test** for the proportions.

The null and alternate hypotheses would be:

$H_0 : p_1 = p_2$

$H_a : p_1 \neq p_2$ or $p_1 < p_2$ or $p_1 > p_2$

Assumptions:

1. Both samples must be independent SRSs from the populations of interest.
2. The population sizes are both at least ten times the sizes of the samples.
3. The number of successes and failures in both samples must all be at least 10.

Recall: $\hat{p}$ = sample proportion (like $\bar{x}$).

p = population proportion

When working with proportions we always use *z* and not *t*.

Test statistic is: $$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}}$$

If p_1 and p_2 are unknown, we will use $\hat{p}_1$ and $\hat{p}_2$ to approximate standard deviation (denominator).

Example 1: Is the proportion of left-handed students higher in honors classes than in academic classes? Two hundred academic and one hundred honors students from grades 6-12 were selected throughout a school district and their left or right handedness was recorded. The sample information is:

	Honors	Academic
Sample size	100	200
Number of left-handed students	18	32

Is there sufficient evidence at the 5% significance level to conclude that the proportion of left-handed students is greater in honors classes?

First, make sure to label the categories above, then for each find:

$n_1 =$

$n_2 =$

$$\hat{p}_1 = \frac{number\ of\ successes}{number\ of\ oberservations} =$$

$$\hat{p}_2 = \frac{number\ of\ successes}{number\ of\ oberservations}$$

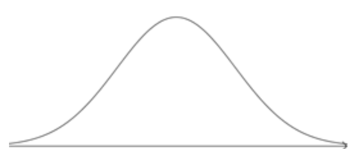
State:

H_0 :

H_a :

$\alpha =$

Is it one-tailed or two-tailed?



Command:

Answer:

Test Statistic:

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}} =$$

Fail to reject the null or reject the null?

Find the p-value:

Conclusion:
Based on 95% certainty (or on a 5% significance level), we'll fail to reject the null which states that the proportions of left-handers in honors and academic classes are not different.

Example 2: North Carolina State University looked at the factors that affect the success of students in a required chemical engineering course. Students must get a C or better in the course in order to continue as chemical engineering majors. There were 65 students from urban or suburban backgrounds, and 52 of these students succeeded. Another 55 students were from rural or small-town backgrounds; 30 of these students succeeded in the course. Test the claim to see if there is a difference between the urban/suburban and rural/small-town success rates at the 5% level.

First, make sure to label the categories above, then for each find:

$n_1 =$ $n_2 =$

$\hat{p}_1 = \dfrac{number\ of\ successes}{number\ of\ oberservations} =$ $\hat{p}_2 = \dfrac{number\ of\ successes}{number\ of\ oberservations}$

State:
H_0 :
H_a :

$\alpha =$ Is it one-tailed or two-tailed?

Command: Answer:

Test Statistic:

$$z = \frac{\left(\hat{p}_1 - \hat{p}_2\right) - \left(p_1 - p_2\right)}{\sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}} =$$

Fail to reject the null or reject the null?

Find the *p*-value:

Conclusion: At a 5% significance level, we reject the null in favor of saying that there is a difference between urban/suburban and rural/small-town students