Section 8.5 Goodness of Fit Test

Suppose we want to make an inference about a group of data (instead of just one or two). Or maybe we want to test counts of categorical data. **Chi-square** (or χ^2) testing allows us to make such inferences.

There are several types of Chi-square tests but in this section we will focus on the goodness-offit test. **Goodness-of-fit** test is used to test how well one sample proportions of categories "match-up" with the known population proportions stated in the null hypothesis statement. The Chi-square goodness-of-fit test extends inference on proportions to more than two proportions by enabling us to determine if a particular population distribution has changed from a specified form.

The null and alternative hypotheses do not lend themselves to symbols, so we will define them with words. These are different from what we're used to seeing.

 H_0 : _____ is the same as _____ H_a : _____ is different from _____

For each problem you will make a table with the following headings:

Observed	Expected	$(O - E)^2$
Counts (O)	Counts (E)	$\frac{E}{E}$

The sum of the third column is called the Chi-square test statistic. Σ represents the sum.

$$\chi^2 = \sum \frac{(\text{observed - expected})^2}{\text{expected}}$$

Chi-square distributions have only positive values and are skewed right. As the degrees of freedom increase it becomes more normal. The total area under the χ^2 curve is 1.



The assumptions for a Chi-square goodness-of-fit test are:

- 1. The sample must be an SRS from the populations of interest.
- 2. The population size is at least ten times the size of the sample.
- 3. All expected counts must be at least 5.

To find probabilities for χ^2 distributions:

R command is: 1 – pchisq(test statistic, df)

Note: degrees of freedom = df = (number of categories -1)

Example: The Mixed-Up Nut Company advertises that their nut mix contains (by weight) 40% cashews, 15% Brazil nuts, 20% almonds and only 25% peanuts. The truth-in-advertising investigators took a random sample (of size 50 lbs) of the nut mix and found the distribution to be as follows:

Cashews	Brazil Nuts	Almonds	Peanuts
15 lb	11 lb	13 lb	11 lb

At the 1% level of significance, is the claim made by Mixed-Up Nuts true?

The following is how you'll normally set up your null and alternate hypothesis.

 H_0 : The data distribution of nuts is the same as the population.

 H_a : The data distribution of nuts is different from the population.

Let's first find the *expected*:

Cashews	Brazil Nuts	Almonds	Peanuts

Next, let's find the test statistic:

 $\chi^2 = \sum \frac{(\text{observed - expected})^2}{\text{expected}} =$

Before we find the p-value, what is df = (number of categories - 1)?

Now find the p-value: