

Section 8.6 Inference for Two-way Tables

We can also use the Chi-Square method to make inferences for data in two-way tables.

The formula to find the expected count in a two-way table is:

$$\text{expected count} = \frac{\text{row total} \times \text{column total}}{n}$$

Where n is the grand total of all values.

When conducting a Chi-square test of independence in a two-way table, the null and alternate hypothesis will be:

H_0 : There is no association between the row and column variables.

H_a : There is an association between the two variables.

The test statistic is:

$$\chi^2 = \sum_{\text{all cells}} \frac{(\text{observed} - \text{expected})^2}{\text{expected}}$$

The p -value: $P(\chi_k^2 > \chi^2)$, where χ_k^2 represents a Chi-square distribution with $df = (r - 1)(c - 1)$.

The assumptions necessary for the test to be valid are:

1. The observations constitutes a simple random sample from the population of interest.
2. The expected counts are at least 5 for each cell of the table.

By itself, the chi-square test determines only whether the data provide evidence of a relationship between the two variables. If the result is significant, one can go on to identify the source of that relationship by finding the cells of the table that contribute most to the χ^2 value (i.e. those cells with the biggest discrepancy between the observed and expected counts) and by noting whether the observed count falls above or below the expected count in those cells.

Example: Use the data below to determine if there is sufficient evidence to conclude that an association exists between car color and the likelihood of being in an accident.

	Red	Blue	White
Car has been in accident	28	33	36
Car has not been in accident	23	22	30

First, let's state our hypothesis.

H_0 : There is **no association** between car color and having an accident.

H_a : There is an **association** car color and having an accident.

Next, we need to calculate our totals:

Observed!

	Red	Blue	White	Row Totals
Car has been in accident	28	33	36	97
Car has not been in accident	23	22	30	75
Column Totals	51	55	66	Grand Total 172

Next, we need the **expected counts**. Recall: $\text{expected count} = \frac{\text{row total} \times \text{column total}}{n}$



	Red	Blue	White
Car has been in accident	$\frac{97(51)}{172} = 28.76$	$\frac{97(55)}{172} = 31.02$	$\frac{97(66)}{172} = 37.22$
Car has not been in accident	$\frac{75(51)}{172} = 22.24$	$\frac{75(55)}{172} = 23.98$	$\frac{75(66)}{172} = 28.78$

The test statistic is:

$$\chi^2 = \sum_{\text{all cells}} \frac{(\text{observed} - \text{expected})^2}{\text{expected}} = \frac{(28 - 28.76)^2}{28.76} + \frac{(33 - 31.02)^2}{31.02} + \frac{(36 - 37.22)^2}{37.22} + \frac{(23 - 22.24)^2}{22.24} + \frac{(22 - 23.98)^2}{23.98} + \frac{(30 - 28.78)^2}{28.78}$$

$$\chi^2 = 0.43$$

Lastly, the p-value:

$$df = (r - 1)(c - 1) = (2 - 1)(3 - 1) = 2$$

$$1 - \text{pchisq}(0.43, 2) = 0.8065 > \alpha = 0.05$$

Fail to reject H_0