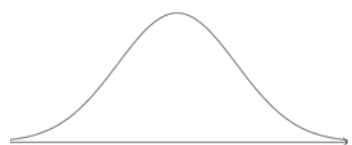
Math 2311
Test 3 Review
17 multiple choice questions. Numbers 1 – 7, 10 points each and numbers 8 – 17, 3 points each.

1. True or False?
   a. The width of a confidence interval narrows as the sample size increase.
   b. The width of a confidence interval widens as the confidence level increases.
   c. Reducing the width of a confidence interval causes the variance or confidence level to decrease.
   d. Increasing the width of a confidence interval causes the variance or confidence level to decrease.
   e. The larger the level of confidence, the shorter the confidence interval.
   f. If we want to claim that a population parameter is different from a specified value, this situation can be considered as a one-tailed test.
   g. In the p-value approach to hypothesis testing, if the p-value is less than a specified significance level, we reject the null hypothesis.



i. In a hypothesis test, if the p-value is less than 0.001 then we fail to reject the null hypothesis.

2. The gas mileage for a certain model of car is known to have a standard deviation of 4 mi/gallon. A simple random sample of 49 cars of this model is chosen and found to have a mean gas mileage of 27.5 mi/gallon. Construct a 96.5% confidence interval for the mean gas mileage for this car model.

Recall: One-sample z-test: $\bar{x} \pm z^* \frac{\sigma}{\sqrt{n}}$

3. A Brinell hardness test involves measuring the diameter of the indentation made when a hardened steel ball is pressed into material under a standard test load. Suppose that the Brinell hardness is determined for each specimen in a sample of size 50, resulting in a sample mean hardness of 64.3 and a sample standard deviation of 6.0. Calculate a 99% confidence interval for the true average Brinell hardness for material specimens of this type.

Recall: One-sample t-test: $\bar{x} \pm t^* \frac{s}{\sqrt{n}}$

4. A simple random sample of 100 7th graders at a large suburban middle school indicated that 86% (86 out of 100) of them are involved with some type of after school activity. Find the 98% confidence interval that estimates the proportion of them that are involved in an after school activity.

Recall: One-proportion z-test: $\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

5. An auditor for a hardware store chain wished to compare the efficiency of two different auditing techniques. To do this he selected a sample of nine store accounts and applied auditing techniques A and B to each of the nine accounts selected. The number of errors found in each of techniques A and B is listed in the table below:

Errors in A	Errors in B
27	13
30	19
28	21
30	19
34	36
32	27
31	31
22	23
27	32

Select a 99% confidence interval for the true mean of the difference in the two techniques. *Let's set this one up only.*

6. The length of needles produced by a machine has standard deviation 1.30 inches. Assuming that the distribution is normal, how large a sample is needed to determine with a precision of ±0.5 (same as "within 0.5") the mean length of the produced needles to 99% confidence?
**When finding the sample size, always use* z^* *(whether proportions are given or not).*

$$z^* \frac{\sigma}{\sqrt{n}}$$

7. The one-sample z statistic for a test of H_0: $\mu = 200$ vs. H_a: $\mu < 200$ based on $n = 10$ observations has the test statistic value of $z = 1.616$. What is the p-value for this test?

8. The one-sample t statistic for a test of H_0: $\mu = 0$ vs. H_a: $\mu > 0$ based on $n = 6$ observations has the test statistic value of $t = 2.162$. What is the p-value for this test?

9. The two-sided t statistic for a test of H_o: $p = 325.16$ vs. H_a: $p \neq 325.16$ based on n = 75 observations has the test statistic $t = -1.453$. What is the p-value for this test?

10. A study of the ability of individuals to walk in a straight line reported the accompanying data on cadence (strides per second for a sample of n = 20 randomly selected men).

.95	.85	.92	.95	.93	.86	1.00	.92	.85	.81
.78	.93	.93	1.05	.93	1.06	1.06	.96	.81	.96

Assuming the standard deviation of the population is 0.08, test the hypothesis that the mean cadence for the population is less than 0.97 at the 5% significance level.

After calculating the mean in R, $\overline{x} = 0.9255$.

a. State the null and alternate hypothesis.

b. Find the rejection region.

c. Find the test statistic.

Recall: $z = \dfrac{\overline{x} - \mu_o}{\dfrac{\sigma}{\sqrt{n}}}$

d. Find the p-value.

e. Conclude: Reject the null or Fail to reject the null

11. Based on information from a large insurance company, 66% of all damage liability claims are made by single people under the age of 25. A random sample of 53 claims showed that 43 were made by single people under the age of 25. Does this indicate that the insurance claims of single people under the age of 25 are higher than the national percent reported by the large insurance company?

a. State the null and alternate hypothesis.

b. Find the rejection region.

c. Find the test statistic.

Recall: $z = \dfrac{\hat{p} - p}{\sqrt{\dfrac{p(1-p)}{n}}}$

d. Find the p-value.

e. Conclude: Reject the null or Fail to reject the null

12. In an experiment to study the effects of illumination level on performance, subjects were timed for completion in both a low light level and high light level. The results are below.

	Subject								
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Low Light	26	29	32	26	21	41	25	25	27
High Light	18	21	23	20	20	25	16	16	25

Can you say with 95% certainty that the average completion time is lower in high light?

After computing the mean and standard deviation in R, $\overline{x_D} = 7.5556$ *and* $s_D = 4.3906$.

a. State the null and alternate hypothesis.

b. Find the rejection region.

c. Find the test statistic.

Recall: $t = \dfrac{\overline{x}_D - \mu_D}{s/\sqrt{n}}$

d. Find the p-value.

e. Conclude: Reject the null or Fail to reject the null

13. A sample of 97 Duracell batteries produces a mean lifetime of 10.40 hours and standard deviation 4.83 hours. A sample of 148 Energizer batteries produces a mean lifetime of 9.26 hours and a standard deviation of 4.68 hours. At a 5% significance level, can we assert that the average lifetime of Duracell batteries is greater than the average lifetime of Energizer batteries?

Duracell Energizer

a. State the null and alternate hypothesis.

b. Find the rejection region.

c. Find the test statistic.

Recall: $t = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$

d. Find the p-value.

e. Conclude: Reject the null or Fail to reject the null

14. A random sample of size 36 selected from a normal distribution with $\sigma = 4$ has $\bar{x} = 75$. A second random sample of size 25 selected from a different normal distribution with $\sigma = 6$ has $\bar{x} = 85$. Is there a significant difference between the two population means at the 5% level of significance?

a. State the null and alternate hypothesis.

b. Find the rejection region.

c. Find the test statistic.

Recall: $z = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$

d. Find the p-value.

e. Conclude: Reject the null or Fail to reject the null

15. In a recent publication, it was reported that the average highway gas mileage of tested models of a new car was 33.5 mpg and approximately normally distributed. A consumer group conducts its own tests on a simple random sample of 12 cars of this model and finds that the mean gas mileage for their vehicles is 31.6 mpg with a standard deviation of 3.4 mpg. Test whether these data cast doubt on the current report.

a. State the null and alternate hypothesis.

b. Find the rejection region.

c. Find the test statistic.

Recall: $t = \dfrac{\overline{x} - \mu}{s / \sqrt{n}}$

d. Find the p-value.

e. Conclude: Reject the null or Fail to reject the null

16. Mars Inc. claims that they produce M&Ms with the following distributions:

Brown	20%	Red	25%	Yellow	25%
Orange	5%	Green	15%	Blue	10%

A bag of M&Ms was randomly selected from the grocery store shelf, and the color counts were:

Brown	25	Red	23	Yellow	21
Orange	13	Green	15	Blue	14

Is this a χ^2 goodness of fit test?

Use $\alpha = 0.05$ to determine if the proportion of M&Ms is what is claimed.

Brown		Red		Yellow	
Orange		Green		Blue	

a. Find the test statistic.

$$\chi^2 = \sum \frac{(\text{observed - expected})^2}{\text{expected}}$$

b. Find the p-value.

c. Conclude: Reject the null or Fail to reject the null

17. Identify the type of test.

a) Matched pairs
b) One sample *t* test
c) Two sample *t* test
d) Two sample *z* test
e) One sample z test

I. It is believed that the average amount of money spent per U.S. household per week on food is about \$96, with standard deviation \$9. A random sample of 49 households in a certain affluent community yields a mean weekly food budget of \$100. We want to test the hypothesis that the mean weekly food budget for all households in this community is higher than the national average.

II. A national computer retailer believes that the average sales are greater for salespersons with a college degree. A random sample of 14 salespersons with a degree had an average weekly sale of \$3542 last year, while 17 salespersons without a college degree averaged \$3301 in weekly sales. The standard deviations were \$468 and \$642 respectively. Is there evidence to support the retailer's belief?

III. Quart cartons of milk should contain at least 32 ounces. A sample of 22 cartons was taken and amount of milk in ounces was recorded. We would like to determine if there is sufficient evidence exist to conclude the mean amount of milk in cartons is less than 32 ounces?

IV. In an experiment on relaxation techniques, subject's brain signals were measured before and after the relaxation exercises. We wish to determine if the relaxation exercise slowed the brain waves.

V. A private and a public university are located in the same city. For the private university, 1046 alumni were surveyed and 653 said that they attended at least one class reunion. For the public university, 791 out of 1327 sampled alumni claimed they have attended at least one class reunion. Is the difference in the sample proportions statistically significant?

VI. An experimenter flips a coin 100 times and gets 43 heads. Test the claim that the coin is fair against the two-sided claim that it is not fair at the level α=.01.

Hypothesis tests:

Test	Null Hypothesis	Test Statistic
One-sample z-test for means	$\mu = \mu_o$	$z = \dfrac{\overline{x} - \mu_o}{\dfrac{\sigma}{\sqrt{n}}}$
One-sample t-test for means	$\mu = \mu_o$	$t = \dfrac{\overline{x} - \mu_o}{\dfrac{s}{\sqrt{n}}}$; df = n–1
Matched Pairs t-test	$\mu_D = \mu_{D_0}$	$t = \dfrac{\overline{x}_D - \mu_D}{s / \sqrt{n}}$; df = n – 1
One-sample z-test for proportions	$p = p_o$	$z = \dfrac{\hat{p} - p}{\sqrt{\dfrac{p(1-p)}{n}}}$
Two-sample t-test for means df=min(n1,n2)-1	$\mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$	$t = \dfrac{\overline{x}_1 - \overline{x}_2}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$;
Two-sample z-test for proportion	$p_1 - p_2 = 0$ or $p_1 = p_2$	

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\left(\dfrac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \dfrac{\hat{p}_2(1-\hat{p}_2)}{n_2}\right)}}$$

χ^2 Goodness of fit test — no change

$$\chi^2 = \sum \frac{(\text{observed} - \text{expected})^2}{\text{expected}}$$

Confidence Intervals

General Formula: statistic $\pm$ margin of error

One-sample z-test: $\bar{x} \pm z^* \dfrac{\sigma}{\sqrt{n}}$

Two-proportion z-test: $(\hat{p}_1 - \hat{p}_2) \pm z^* \sqrt{\dfrac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \dfrac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$

One-sample t-test: $\bar{x} \pm t^* \dfrac{s}{\sqrt{n}}$

One-proportion z-test: $\hat{p} \pm z^* \sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}$

Two-sample z-test: $(\bar{x}_1 - \bar{x}_2) \pm z^* \sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}$

Two-sample t-test: $(\bar{x}_1 - \bar{x}_2) \pm t^* \sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}$