MATH 3336 – Discrete Mathematics
Strong Induction (5.2)

Principle of Strong Mathematical Induction: To prove that $P(n)$ is true for all positive integers n, we complete these steps:

- *Basis Step*: Show that $P(1)$ is true.
- *Inductive Step*: Show the conditional statement $[P(1) \wedge P(2) \wedge \dots \wedge P(k)] \rightarrow P(k+1)$ is true for all positive integers k.
- Inductive Hypothesis includes ALL k statements $P(1), P(2), \dots, P(k)$.

Example: Every positive integer greater than 1 can be expressed as a product of primes.
Proof:

Basis step: $P(\quad)$

Inductive step: $[P(1) \wedge P(2) \wedge \dots \wedge P(k)] \rightarrow P(k+1)$

Conclusion: By the principle of ***strong*** mathematical induction, the statement is true for all integers greater than 1.

Example: Prove that every amount of postage of 12 cents or more can be formed using just 4-cent and 5-cent stamps.

Basis step: $P(\quad)$

Inductive step: If $P(j)$ for $12 \leq j \leq k$, then $P(k+1)$.

Conclusion: By the principle of ***strong*** mathematical induction, the statement is true for all integers greater than 12.

Example: Let $f_n = f_{n-1} + f_{n-2}$, $f_0 = 1$, and $f_1 = 1$. Show that $f_n \leq 2^n$ for all positive n.

Proof:

Basis step: $P(\quad)$

Inductive step: If $P(j)$ for $1 \leq j \leq k$, then $P(k+1)$.

Conclusion: By the principle of ***strong*** mathematical induction, the statement is true for all positive integers.