MATH 3336 – Discrete Mathematics
Generalized Combinations and Permutations (6.5)

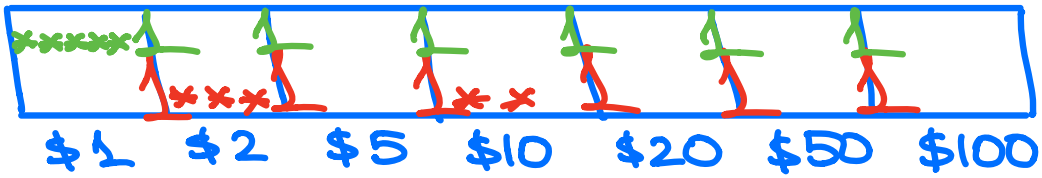
Permutations *with Repetitions*
Theorem: The number of r-permutations of a set of n objects with repetition allowed is n^r.

Example: How many strings of length 5 can be formed from the uppercase letters of the English alphabet?

26^5

Combinations *with Repetition*

Example: How many ways are there to select five bills from a box containing at least five of each of the following denominations: \$1, \$2, \$5, \$10, \$20, \$50, and \$100?



three \$2
+ two \$10] 1 * * * 1 1 * * 1 1 1

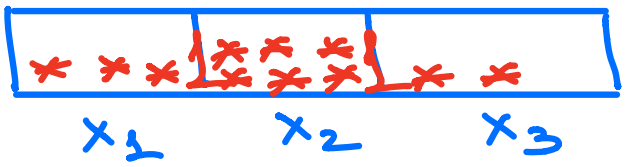
C(11,6)
= C(11,5)

five \$1] * * * * * 1 1 1 1 1 1

Theorem: The number of r-combinations from a set with n elements when repetition of elements is allowed is

$$C(n+r-1,r) = C(n+r-1,n-1)$$

Example: How many solutions does the equation $x_1 + x_2 + x_3 = 11$ have, where x_1, x_2, and x_3 are nonnegative integers?



$x_1 = 3$
$x_2 = 6$
$x_3 = 2$

$C(13,2)$
$= C(13,11)$

Example: Suppose that a cookie shop has four different kinds of cookies. How many different ways can six cookies be chosen?

$C(9,6) = C(9,3)$

Summarizing the Formulas for Counting Permutations and Combinations with and without Repetition

Permutations and Combinations with and without Repetition	
r −permutation without repetition	$\frac{n!}{(n-r)!}$
r −combination without repetition	$\frac{n!}{r!\ (n-r)!}$
r −permutation with repetition allowed	n^r
r −combination with repetition allowed	$\frac{(n+r-1)!}{r!\ (n-1)!}$

Permutations with *Indistinguishable* Objects

Example: How many different strings can be made by reordering the letters of the word *SUCCESS*?

$S_1 U C_1 C_2 E S_2 S_3$ $\quad \frac{7!}{2!\ 3!}$

Theorem: The number of different permutations of n objects, where there are n_1 indistinguishable objects of type 1, n_2 indistinguishable objects of type 2… and n_k indistinguishable objects of type k, is: $\frac{n!}{n_1!\, n_2! \ldots n_k!}$.