Math 3336 Test 2 Review Questions

What is covered: 2.3, 2.4, Chapter 4 (Sections 1-4), 5.1, 5.2.
How to study: Study the class notes, review homework problems, and try to do as many exercises as you can from the textbook. Note that answers are provided at the back of the book to all odd numbered problems.
Here I provide some examples for you. This is not a complete list, studying only these examples is not enough!

1. Use the Euclidean Algorithm to find $gcd(580, 50)$.

2. Express $gcd(84, 18)$ as a linear combination of 18 and 84.

3. Find an inverse of 5 modulo 12.

4. Solve the linear congruence $5x \equiv 3 \ (mod\ 11)$.

5. Use Fermat's little theorem to find $9^{45}\ mod\ 23$.

6. Find the integer a such that

   a. $a \equiv -15(mod\ 27)$ and $-70 \leq a \leq -50$
   b. $a \equiv 99(mod\ 41)$ and $100 \leq a \leq 140$

7. Decide whether each of these integers is congruent to 5 modulo 17.
   a. 56 b. 12 c. -29

8. Which positive integers less than 12 are relatively prime to 12?

9. Use the Principle of Mathematical Induction to prove that for all positive integers
$$1^2 + 2^2 + \cdots + n^2 = n(n+1)(2n+1)/6.$$

10. Prove that for every positive integer n ,
$$1 \cdot 2 + 2 \cdot 3 + \cdots + n(n+1) = n(n+1)(n+2)/3$$

11. Use the Principle of Mathematical Induction to prove that $n^3 > n^2 + 3$ for all $n \geq 2$.

12. Prove that $2^n > n^2$ if n is an integer greater than 4.

13. Use the Principle of Mathematical Induction to prove that $3|(n^3 + 3n^2 + 2n)$ for all $n \geq 1$.

14. Prove that $n^2 - 1$ is divisible by 8 whenever n is an odd positive integer.

15. Use the Principle of Mathematical Induction to prove that any integer amount of postage from 18 cents on up can be made from an infinite supply of 4-cent and 7-cent stamps.

    Do you use the Principle of Mathematical Induction or the Strong Principle of Mathematical Induction to prove this result?

16. A sequence $a_1, a_2, a_3, \ldots$ is defined recursively by $a_1 = 1, a_2 = 4$ and $a_n = 2a_{n-1} - a_{n-2} + 2$ for $n \geq 3$.
    Conjecture a closed formula for a_n and prove the formula using the Principle of Mathematical Induction.

    Do you use the Principle of Mathematical Induction or the Strong Principle of Mathematical Induction to prove this result?

17. Suppose $f: \mathbb{Z} \to \mathbb{Z}$ has the rule $f(n) = 3n^2 - 1$. Determine whether f is 1-1.

18. Suppose $f: \mathbb{Z} \to \mathbb{Z}$ has the rule $f(n) = 3n^2 - 1$. Determine whether f is onto $\mathbb{Z}$

19. Suppose $g: A \to B$ and $f: B \to C$ where $A = \{1, 2, 3, 4\}$, $B = \{a, b, c\}$, $C = \{2, 7, 10\}$, and f and g are defined by $g = \{(1, b), (2, a), (3, a), (4, b)\}$ and $f = \{(a, 10), (b, 7), (c, 2)\}$. Find $f \circ g$.

20. Suppose $g: A \to B$ and $f: B \to C$ where $A = \{1, 2, 3, 4\}$, $B = \{a, b, c\}$, $C = \{2, 7, 10\}$, and f and g are defined by $g = \{(1, b), (2, a), (3, a), (4, b)\}$ and $f = \{(a, 10), (b, 7), (c, 2)\}$. Find f^{-1}.

21. Find a formula that generates the following sequence $\{1, 1/3, 1/5, 1/7, 1/9, \ldots\}$.

22. Describe the following sequence recursively. Include the initial condition and assume the sequence begins with a_1. $\{1/2, 1/3, 1/4, 1/5, \ldots\}$.